A SURVEY OF THE UNDERSTANDINGS OF SELECTED CONCEPTS OF LOGIC BY 8-18-YEAR-OLD STUDENTS

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TO

MY PARENTS

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KEY TO ABBREVIATIONS

Abbreviation	Definition
A	Age Group
E	Socioeconomic Group
I	Intelligence Group
I.Q.	Intelligence Quotient
S	Sex Group

CHAPTER I

INTRODUCTION

General Background of the Problem

William O. Stanley asserts that there is now no cultural pressure to believe in most given ideas [53];* he concludes that the core of our culture has become smaller. Neither religion nor any other force influences us to accept or to reject very many beliefs. There is now a greater need than in the past for students in our society to form their own beliefs. This is a need, which has resulted from the decrease in the number of core values of our culture, to evaluate existing factual knowledge for the purpose of forming one's own value system.

The research and other efforts of scholars, inventors, and other specialists throughout the world as well as the products of technical advancements have resulted, however, in what some writers call a "knowledge explosion"; their claim is that there has been an increasingly rapid growth in the amount of man's factual knowledge. In Readings in Curriculum by Glen Hass and Kimball Wiles, Bentley Glass wrote that

> . . . from 7000 B.C. until the invention of printing--in just a few thousand years--there was far more change in the nature of man's existence and his control over his environment than in the half-million years before. From 1600 to 1900, science and technology have produced far more change in the nature of man's life

*Numbers in brackets refer to the Bibliography.

> than occurred in the preceding 6000 years. We live far more differently today than people in 1600 lived differently from those in 7000 B.C. . . .
>
> And we still experience the logarithmic effect in the increase of scientific knowledge. [15:273]

Saylor and Alexander wrote about a knowledge-generation, the span of time during which the total body of what is known, understood, and used doubles; the authors claim that a knowledge generation is now just ten to twenty-five years. "No person living today can hope to know many things of great importance in human affairs, nor can he be a specialist in more than a quite limited aspect of human life," they claim [45:107].

The dilemma facing the curriculum planner is apparent. What knowledge from this vast reservoir should be selected for students to study? What knowledge is of most worth? What knowledge will prepare students best for their futures? Since many people believe that no group today appears to know, it is important to teach students how to answer these questions for themselves. Each of them must sift through the vast amount of factual knowledge which presents itself to them; they must analyze facts and interconnect them in order to form generalizations which will be the bases for their system of beliefs.

The selecting and processing of factual knowledge may involve the use of logic, the science of formal principles of reasoning. Because of the knowledge explosion, logic is more important today than it has been in past centuries. It can be used to teach students, as far as it is possible, to think clearly. Assuming that a student who can think clearly will be prepared to tackle the jobs of the future and to cope

with novel life situations which he will face with a set of values which he himself has developed and in which he himself believes.

Background Related to Mathematics

"A satisfactory study of thinking in any field . . . requires a detailed consideration of the reasoning process itself" [51:43]. The primary reasoning process of mathematics is deductive logical reasoning, by which is meant "the process used in deducing new statements or propositions from one or more given statements" [51:43]. A survey of present understandings of logical reasoning in mathematics is basic to an understanding of the thinking done when studying mathematics and, hence, of the teaching and learning of mathematics.

A study of the thinking done when studying mathematics may have applications in other fields also. After reviewing 196 references, Pedro T. Orata concluded from 211 studies that the transfer of training "is a fact, as revealed by nearly 80 percent of the studies" which were done between 1890 and 1940 [32:81]. Studies by Thorndike and Woodworth, Katona, and Judd, perhaps the most widely read studies on transfer of training, are among those summarized in the twenty-first Yearbook of the National Council of Teachers of Mathematics; each of them support the theory of transfer of training [41]. Becker and McLeod also summarize research which supports the theory of transfer of training [3].

"Any mathematics course in school or college should aim to give pupils as large an appreciation of the nature of mathematics as possible" [51:30]. But logic and mathematics are closely related so that a knowledge of logic will help one in achieving some idea of the nature of

mathematics. "That mathematics is in some way intimately bound up with logic has been recognized since the days of Euclid, who, without explicit formulation, utilized the laws of logic in many of his . . . proofs" [25:99]. Since the study of logic can give students better understandings of the nature of proof and, therefore, of the nature of mathematics, logic should be a concern of mathematics educators.

In past decades, "logic has gained depth and breadth as a subject for study. Only in recent years have the systematic relations between logic and mathematics been established and a completely explicit theory of inference formulated which is adequate to deal with all the standard examples of deductive reasoning in mathematics and the empirical sciences" [55:58]. According to Donovan Johnson, we now emphasize logic, proof, and reasoning in mathematics [21:418; 47 and 52].

Many authors in the field of mathematics education see the need for work in symbolic logic in schools [elementary: 1; secondary: 22, 29 and 42]. Some of the authors believe that some logic ought to be included as a unit in mathematics courses; such units are now included in some new high school textbooks. Chapter two of Concepts of Modern Mathematics, Book 2, by Morton R. Kenner, Dwain E. Small, and Grace N. Williams, for example, deals with logic [23]. Others feel that logic could and should be taught as a separate mathematics course. An Algebra II class at University High School of Florida State University [54], for example, studied symbolic logic and designed and built electric circuits to illustrate conjunction, disjunction, implication, double implication, negation, and equivalence. The class consisted of just six students.

Brown [5 and 6] and Giles [14] also considered construction and use of electrical devices for supplementary work and/or to encourage drill in logic.

Many authors [e.g., 57:282; 2, 18, 42 and 46] are in agreement that the following topics of logic ought to be taught in the mathematics classroom: quantifiers, truth values of conjunctions and disjunctions, negations of simple as well as compound statements, conditional statements as well as their converses, inverses, and contrapositives, and an analysis of the nature of proof. Many articles have been written on these aspects of logic. Nathan Lazar wrote classic articles on converses, inverses, and contrapositives of statements, for example [25, 26 and 27]. Tierney [58] wrote about conjunction and disjunction. Dupree [11], May [28], Rourke [43], and Santner [44] dealt with various aspects of implication; Chachere [8] and Dupree [11] were concerned with contrapositives.

Statement of the Problem

If the teaching of mathematics is to be improved, the thinking processes used in mathematics should be more fully understood and a determination should be made as to when and how students understand the aspects of logic which are used in mathematics. This study is directed toward the problem of determining how well various groups of students from ages 8 to 18 understand certain aspects of deductive logic which are used in mathematics. The aspects of logic which have been investigated include quantifiers, conjunctive and disjunctive statements,

implications, and types of valid and invalid inferences. Specific purposes are to determine as far as possible:

A. Level of students' understandings of

1. the validity of conclusions derived from four types of commonly-used inferences: modus ponens, modus tollens, affirmation of conclusion, and denial of hypothesis;

2. the truth values of

a. compound propositions (conjunctive, disjunctive, and conditional);

b. converse, inverse, and contrapositive propositions;

c. propositions with the quantifiers "all," "any," "each," "every," and "some."

B. The statistical significance of differences in understandings of logic for students of different ages, sexes, intellectual abilities, and socioeconomic status.

C. The statistical significance of interaction between combinations of students' age, I.Q., sex, and/or socioeconomic level upon understandings of logic.

D. The statistical significance of differences in variation of understandings of type of logic for students of various age groups.

Hypotheses

The following are considered as hypotheses because of their importance in mathematics curriculum planning and general curriculum planning:

1. When students of different age groups are compared, the older students will have a more complete understanding of deductive logic.
2. When students of different ability levels are compared, those of the higher levels will have a more complete understanding of deductive logic.
3. There will be no difference between the understandings of deductive logic for the boys and girls.
4. When students of different socioeconomic levels are compared, those of the higher levels will have a more complete understanding of deductive logic.
5. When students of various combinations of the variables (intelligence, age, sex, and socioeconomic level) are compared, differences in understandings of deductive logic will be found.
6. There will be more variability within the scores of those of ages eleven through thirteen than within the scores of those of ages younger than eleven or the scores of those older than thirteen.

Definition of Terms

The following are definitions and examples of some relevant terms:

Proposition: a statement which is capable of being affirmed or denied. A proposition may be represented by a letter of the alphabet.

Proposition p: John is a democrat.
Proposition q: Paul is a republican.

Compound proposition: a proposition consisting of two simpler propositions joined by one or more connecting words such as "and," "or," or "if ..., then ...."

John is a democrat and Paul is a republican.
John is a democrat or Paul is a republican.
If John is a democrat, then Paul is a republican.
p and q. p or q. If p, then q.

Conditional proposition (implication): a compound proposition which can be put into the form "If p, then q"; p is called the hypothesis and q, the conclusion.

If John is a democrat, then Paul is a republican.

Negation: denial of a proposition; the contradiction of a proposition.

Proposition:	p	John is a democrat.
Negation:	not p	John is not a democrat.

Converse of a conditional proposition: a conditional proposition obtained by putting the hypothesis of the original conditional proposition in place of its conclusion and vice versa.

Proposition:	If p, then q.
Converse:	If q, then p.

Inverse of a conditional proposition: a conditional proposition obtained by taking the negation of the hypothesis and the negation of the conclusion of the original conditional proposition.

Proposition:	If p, then q.
Inverse:	If not p, then not q.

Contrapositive of a conditional proposition: a conditional proposition formed by taking the negations of the hypothesis and conclusion of a given conditional proposition, then putting the resultant hypothesis in place of the resultant conclusion and vice versa.

Proposition:	If p, then q.
Contrapositive:	If not q, then not p.

True: conformable to fact; correct.

Grass is green.

Valid: based on laws of logic; sound; well grounded.

Modus ponens: an argument which can be put into the form "If p, then q. p. Therefore q." Such an argument is always valid.

If John is a democrat, then Paul is a republican.
John is a democrat. Therefore Paul is a republican.

Modus tollens: an argument which can be put into the form "If p, then q. Not q. Therefore not p." Such an argument is always valid.

If John is a democrat, then Paul is a republican.
Paul is not a republican. Therefore John is not a democrat.

Conjunctive proposition: a compound proposition formed from two simpler propositions linked by the word "and." Such propositions are true if and only if both of the simpler propositions are true.

p and q.
John is a democrat and Paul is a republican.

Disjunctive proposition: a compound proposition formed from two simpler propositions linked by the word "or." Such propositions are true if and only if at least one of the simpler propositions is true.

p or q.
John is a democrat or Paul is a republican.

Quantifiers: words such as all, any, each, every, and some, which modify or qualify with respect to quantity.

All: complete; entire; the whole; each; every.

Any: not none; at least one.

Each: complete; entire; the whole; every; all.

Every: complete; entire; the whole; each; all.

Some: not none; at least one; a certain unknown or unspecified quantity.

CHAPTER II

REVIEW OF THE LITERATURE

Review of Research

Fourteen experimental studies related to aspects of mathematical logic are summarized below.

In 1961-1962, Patrick Suppes and Frederick Binford [56; in 16] attempted to teach logic to bright fifth and sixth graders in the San Francisco Bay area. Over 800 students were chosen on the basis of high scores on an arithmetic test (i.e., upper 25%) to take part in their study. They were taught logic three days per week for 30-35 minutes each day for an entire school year. One hundred eighty-six students in a class in logic at Stanford University served as a control group. Two tests were administered to the students, each test after the students had studied given amounts of the logic course content. Conclusions were that (a) the upper quarter of elementary school students can achieve a significant conceptual and technical mastery of elementary mathematical logic, (b) elementary students need more direct teacher supervision and a longer period of time to learn logic than do college students, (c) "dedicated and able elementary teachers" can be trained in five to six semester hours to teach classes in elementary logic, and (d) there is some carryover of logic into other areas of mathematics and into other subjects (ex., reading and English).

Smith [48] was very critical of Suppes and Binford's research. He first noted that their grade-school sample was not the upper quarter of fifth and sixth graders, but a much smaller percentage of them; according to I.Q.'s, 75% of their sample was in the top 7% of the population at this grade level and 50% of their sample was in the top 3% of the population at this grade level. Suppes and Binford wrote that "it is fair to assume that the native ability of the elementary school children selected for the program is comparable to that of the Stanford University students" [56:190]. Smith felt that it is highly unlikely that half of the students at Stanford were in the upper 3% of the population with respect to native intellectual ability. Smith's second criticism was that only about 47% of the elementary students who began the course in logic completed both tests. It is not impossible that students whose scores were considered to be undesirable were not included in the report of the experiment; in such a case, conclusions would be much less convincing. Smith also noted that Suppes and Binford neglected to include many details in their report; they did not mention how they chose the "dedicated and able teachers," for example. Perhaps they were just teachers who volunteered to take five or six credit hours of work at Stanford and to help Suppes and Binford; maybe they were labeled "dedicated and able" in order to reinforce their efforts in this experiment.

George H. Hyram [20] tried to develop critical thinking in children in the upper grades of elementary school. He taught 200 children for 250 minutes per week for four months in St. Louis, Missouri.

Course content included the nature of thinking in general, the tools of thinking, the nature of definition, the nature of deductive inference, the nature of experimentation, and common errors in reasoning. In 1957, he reported that (a) correct (logical) thinking does not depend upon a knowledge of the principles of logic and (b) principles of logic can be used as learning content.

Kenneth A. Retzer and Kenneth B. Henderson did research on the "Effect of Teaching Concepts of Logic on Verbalization of Discovered Mathematical Generalizations." They had eighty junior high school students study a unit Sentences in Logic, then tested the group and a control group on their abilities to verbalize universal generalizations. Both the experimental and the control group were divided into two groups: (a) those of I.Q. 135 and up, and (b) those of I.Q. 116-128. They concluded that "the subjects who studied the Sentences in Logic unit did significantly better than the control group in verbalizing universal generalizations precisely; the gifted students verbalized significantly better (0.005 level) than the other college-capable group" [40; 39:709].

Sr. Mary de Lourdes has recently finished work on her doctoral dissertation, "A Study of Logical Thinking in Elementary School Children, Grades 3 and 6." "This research study proposes to design certain teaching strategies and lessons which offer children opportunities to manipulate logical ideas such as class inclusion, class exclusion, implication, and conjunction in a way that is coordinated with and cognizant of their maturity, experience, and level of mathematical achievement" [9:6]. She hoped to test eight hypotheses by having logic taught

(a) when interwoven with mathematics, and (b) apart from mathematics. Teachers in groups (a) and (b) received in-service education in logic. There was also (c) a class which was not taught logic, but whose teachers got in-service education in logic, and (d) a control group where neither teachers nor students were taught logic. Students were from the Detroit area and were of average ability. Sr. de Lourdes was hoping for a publication date at the University of Michigan of no later than December, 1968.

Bruner, Goodnow, and Austin did research on teaching logical relationships such as conjunction and disjunction, using different colored cards with different numbers of triangles, squares, and circles on them [7]. From their study of Harvard University students, they found that disjunctive concepts were difficult to learn [7 :41]. Dienes [10] did similar research on conjunction and disjunction.

Sigfried Haenisch did "A Study of the Place of Logic in an Elementary Algebra Course" at Rutgers University in 1967 [17]. She read related literature, then wrote an outline for a course in elementary algebra which synthesized logic. The outline included units on the real number line, subsets of the real number field, truth sets of open sentences in one variable, relations, and functions. The central part of her study was the writing of the unit. Miss Haenisch taught the unit to a Highland Park, New Jersey, eighth-grade class for four weeks, however, and gave the thirty-one-member class a sixty-item multiple-choice algebra test which required a knowledge of conjunction, disjunction, negations, implications, equivalences, and the law of

detachment; all of the students got 75% or more items correct on her test.

In 1965-1968, Howard K. Smith did research at Arizona State University on "The Effects of Instruction in Set Theory Upon the Logical Reasoning of Seventh-Grade Students and Subsequent Effects Upon Their Learning to Solve Percentage Problems." "The purpose of this study was to discover whether students subjected to instruction in set theory did, in fact, develop a superior reasoning skill which (a) facilitated their later progress in mathematical study, and (b) proved advantageous to them also in their solving of reasoning problems outside of the field of mathematics" [49:4]. Smith taught two seventh-grade classes set theory for forty minutes each of twenty consecutive days. Two similar control classes worked from the book published by Scott, Foresman, Seeing Through Mathematics, Book I, during the time. Both groups were given a pre-test and a post-test. "The results of the . . . experiment with reference to gains made by the experimental group in logical reasoning ability made it possible to conclude that, insofar as the tests used to measure logical reasoning ability are valid, the seventh-grade students . . . who were taught set theory did in fact gain significantly more in their ability to reason logically than those students who were subjected for a like period to traditional review" [49:115-116]. Neither logical ability nor I.Q. significantly affected computational skill or problem solving ability at the 0.05 level of confidence. Both slow and fast students profited from the instruction in set theory.

After examining elementary, secondary, and college mathematics books and other related literature, Harold W. Brockman did "A Critical

Study of the Use of the Terms 'Necessary Conditions' and 'Sufficient Conditions' in the Teaching of Mathematics" at Ohio State University in 1962 [4]. He defined "necessary conditions" and "sufficient conditions," developed a test of understandings of the two concepts and administered it to 187 graduate students and teachers (only a few of whom had had a logic course) at Ohio State University and Capital University, and reported aids which he found to help in understanding the two types of conditions. From the results of his tests, he concluded that many of those tested were unfamiliar with necessary conditions, sufficient conditions, converses, inverses, and/or contrapositives, and that textbooks do not help in understanding necessary conditions or sufficient conditions. He recommended that students be introduced to the two types of conditions in elementary school and that students use what they learn in all future mathematics courses. He concluded that there should be a big push for understanding of necessary conditions and sufficient conditions in high school geometry courses.

"The Effect of the Use of Mathematical Logic in High School Geometry: An Experimental Study" was the concern of John L. Platt at Colorado State College in 1967 [37]. "The purpose of this experimental study was to examine a means of teaching deductive methods in high school geometry. Specifically, the problem was to evaluate the effect of the use of mathematical logic in high school geometry on (1) achievement of students in geometry, (2) achievement of students in reasoning in geometry, (3) critical thinking of students, and (4) attitude of students toward logic, deduction, and proof in mathematics. Secondarily, the purposes included evaluation of the relative effects on three

achievement levels of students and upon the two sexes" [37:iv]. Platt taught twelve geometry classes in Arapahoe County, Colorado, a four-week unit on mathematical logic and applications of the logic in learning deductive methods in geometry. The unit dealt with simple connectives and their symbolism, truth values of simple connectives and their symbolism, truth values of sentences involving simple connectives, tautologies, and common rules of inference, analyses of arguments, and proof. Platt's conclusions were that (1) the teaching of logic is appropriate for average and high ability students and can be successfully taught by typical secondary school mathematics teachers ("The logic is both enjoyable and challenging to most students at all achievement levels" [37:vi]), (2) there is no loss of achievement in geometry because of time spent on logic, (3) classes taught logic were not "significantly superior to the traditional course in its overall effect upon student achievement in reasoning in geometry, critical thinking ability, or attitude of students toward logic, deductive thinking, and proof in mathematics" [37:vi], and (4) the teaching of logic was more effective with boys with respect to critical thinking. An experimental group consisted of one class of high-achieving students who had two years of algebra, two classes of high-achieving students who had one year of algebra, and three classes of average-achieving students who had one year of algebra. Six similar control classes were taught traditional geometry for the four weeks.

The above studies deal with the ability of students to learn some concepts of logic; they all indicate that logic can be taught.

The next four studies deal with the age at which children develop some concepts of logic.

Four Closely Related Studies

Shirley Hill, in her doctoral dissertation, examined "the abilities of first, second, and third grade children to derive valid inferences from sets of verbal premises" and to consider the relationship of certain factors to this skill [19:1;31]. In 1961, she administered a 100-item yes-no test to 270 children; the test included "sentential logic, with both simple and compound tautological implications, classical syllogisms, and simple quantification logic, with both universal and existential quantifiers." She concluded that "children in the age range of six years through eight years are able, to a marked degree, to recognize valid conclusions derived from hypothetical premises" [19:80]. Scores on her test increased with age, but there were no fixed stages in development. Difficulties with specific topics in logic varied with age. Sex differences were not significant.

In 1967-1968, William A. Miller studied understandings of six logical inference patterns by eighth-, tenth-, and twelfth-grade students in four school systems of southeastern Wisconsin. Miller wrote a test of six logical inference patterns: law of detachment, contrapositive inference, hypothetical syllogism, disjunctive syllogism, affirming the consequent, and denying the antecedent. By administering the test and then discussing it with classes, he found that "(1) students in all three grades were able to select the valid conclusions to the valid

inference patterns law of detachment and contrapositive inference, (2) students in the three grades did not recognize the two invalid patterns as invalid, but rather accepted them as valid, [and] (3) eighth grade students above the 72nd percentile in mental ability . . . scored about as well as any group on the test or on any subtest" [30: 548-549]. In 1968, Miller also did a study with a seventh-grade class in Whitewater, Wisconsin. He taught twelve fifty-minute classes on logical inference patterns and found that the students could learn and enjoy the material. No statistical evaluation was done.

Piaget did considerable work in logic with individuals twelve years of age and younger. Flavell goes so far as to question whether there are any Piaget experiments which did not have to do with logic [13:303]. Piaget worked with implications, quantification, and compound statements and found that whereas students use the words of logic at a very early age, "development of capacity for 'hypothetical' reasoning or formal aspects of logic begins about age eleven" [36:189; 33, 34,and 35]. In Six Psychological Studies, he noted that "at around eleven to twelve years new operations appear by means of the progressive generalization of preceding operations and become stabilized at around fourteen to fifteen years. These are the operations of 'logical propositions,' which, from this time on, can deal with simple verbal statements (propositions), i.e., with simple hypotheses rather than exclusively with objects. Hypothetico-deductive reasoning thus becomes possible and with it the constitution of a 'formal' logic, i.e., a logic applicable to any kind of content" [35:125]. Again, in Psychology of Intelligence,

Piaget wrote that "from 11-12 years and during adolescence, formal thought is perfected and its groupings characterize the completion of reflective intelligence" [34:123]. Although Piaget found that the ability to understand certain concepts of logic began at age 11-12, he did not hold rigidly to any set age for all individuals; social factors, for example, can affect the age at which an individual is capable of logical thought [34: Chap. VI].

From May, 1962, to September, 1964, Robert H. Ennis did a study entitled "Critical Thinking Readiness in Grades 1-12" at Cornell University. "Primary goals of the project were to contribute to a knowledge about what critical thinking is and to a knowledge about when it can be taught" [12:1]. Ennis wrote a seventy-two item yes-no-maybe test on conditional logic and class logic. There were

> 803 subjects distributed in grades 4-12 (inclusive . . . in this study. . . . At each grade level one class-size group was selected for purposes of instruction in logic. There were 217 of these subjects . . . who by design were intended to be fairly representative of the upper New York State school system from which they were drawn. From the same school system comparable classes were selected, again one at each grade level. These subjects were 211 in number. . . . From another upper New York State school system another 375 students (roughly two class-size groups per grade level) judged representative of the school system from which they were drawn . . . were selected for use. [12:7].

There were six twelve-item parts to the test, each on a different type of logic. Some of Ennis's conclusions are the following:

> 1. The basic principles of class and conditional logic can be formulated with sufficient precision to provide a basis for logic tests.
> 2. Piaget's notion of logic deviates to some extent from ours and from other popular interpretations, but is similar enough to warrant a judgement that he and we are basically talking about the same thing. . . .

5. There is a fairly gradual overall improvement in knowledge of logic from age 10-18 among the students we studied who were not given any special training in logic. This improvement holds for the logical fallacies and transitivity, but not for contrapositions, for which there was little improvement. . . .

7. The basic principles of neither class nor conditional logic are generally mastered by age 11-12--among our students not receiving special instruction in logic. . . .

11. We did not find evidence for the existence of stages in development of knowledge of logic among students not receiving special training. . . .

17. No relationship was found between sex and knowledge of logic.

18. Significant but fairly small correlations were found between socioeconomic status and knowledge of logic (.20 for conditional logic and .26 for class logic).

19. Significant and medium-size correlations were found between IQ score and knowledge of logic (.58 for conditional logic and .52 for class logic).

20. Correlations between knowledge of logic and chronological age (over the range 10-18) were significant and medium in size (.58 for conditional logic and .68 for class logic).

21. The relationship between knowledge of logic and chronological age at each grade level (grade 4-12) were generally not significant and tended more toward being negative than positive.

22. Since the students in this study were above average in intelligence, these findings should not be extrapolated without further investigation to students who are below average. [12:25-28]

Summary

Interest in the part logic plays in school mathematics has increased tremendously in just the past ten years. Writers and researchers now agree that logic has considerable value in and for mathematics; many think it should be taught as a course unit. Little research has been done on the relationship of logic to mathematics. It has been shown that logic can be taught to some children; only three studies have been done to determine when students develop understandings

of the concepts of logic. Shirley Hill concluded that "children in the age group six through eight years are able to a marked degree, to recognize the validity of logical inferences" and that test scores increased with age, but there were no fixed stages in development. Miller found that eighth-, tenth-, and twelfth-grade students can recognize valid conclusions to valid inference patterns. Piaget concluded that "development of capacity for 'hypothetical' reasoning or formal aspects of logic begins about age eleven." Ennis concluded that "the basic principles of neither class nor conditional logic are generally mastered by age 11-12--among our students not receiving special instruction in logic." This study was intended to determine if all children eleven years old and older have more ability to understand selected basic types of logic than do those under eleven years of age.

This study was larger in scope than Piaget's, Miller's, or Hill's studies. Children of grades 4-12 were included in this study; sample size of this study was 860. This study was concerned with a more limited area of logic than was Ennis's study; however, this study dealt with students of high, medium, and low abilities. Type of test and types of logic were different in this study from any of the four studies.

CHAPTER III

THE DEVELOPMENT OF THE TEST INSTRUMENT

Data were collected mainly through the use of a two-part, written, multiple-choice test designed and administered by the researcher. The first part of the test consisted of questions involving implications and the second part of questions involving truth values of propositions. Questions dealt with very common everyday objects and/or events. There were five choices per item.

Since the test instrument was the basis of this study, it was essential that it be valid. To ascertain test validity, the test instrument (1) was read by a jury of experts and revised on the basis of their comments, (2) was shown by means of a pilot study to have a high degree of reliability, clear directions, and a high probability that students' scores could not be achieved by relying entirely on guessing, (3) was shown to have a low reading level with few unfamiliar words, and (4) was shown by means of computer programs to be valid and reliable. This chapter describes the way in which these checks of test validity were made.

Jury Judgement

The panel of judges who read and criticized the test consisted of the following eleven people: Dr. Kenneth P. Kidd, leader in the

field of mathematics education; Dr. Betty Plunkett, assistant professor of (mathematics) education; Dr. Nicholas Vigilante, associate professor of mathematics and education; Dr. Waldemar Olson, professor of education; Mr. Thomas E. Massey, secondary mathematics teacher; Mrs. Bennett R. Ford, fourth-grade teacher; Mr. Ed Davis, secondary mathematics teacher and doctoral student in mathematics education; Miss Cherry D. Ford, mathematics and logic teacher at the junior college level; and Mr. Ben Cheatham and Mr. Bill Palow, doctoral students in mathematics education and teachers of extension courses in mathematics at the university level. The jurists were all from schools in Gainesville, Florida. Each of them was sent a letter explaining the study and asking him for his help in judging the test. Each of them indicated his willingness to serve on the jury.

To keep the number of communications with the jury at a minimum, more than twice as many questions were written as were needed (Column 1, Table 1; Appendix 1). Numerous logic books, pamphlets, and work-sheets had been examined to get ideas for eighty multiple-choice questions dealing with the eight types of logic with which this study was concerned. Test items were sent to jury members, who were asked to read each question, then to (1) circle ambiguous words, phrases, or questions, (2) write the words "too easy" and/or "too hard" next to questions which were too easy for twelfth graders and/or too hard for fourth graders, respectively, (3) put an X in front of any questions which were judged not to test what they were designed to test, (4) make any additional comments which they may have wanted to make next to

TABLE 1

DEVELOPMENT OF TEST ITEMS

Category	Number of Items of Jury Test	Items of Jury Test Which Were Used For		
		Pilot Test	Final Test	Data Analysis
Part I of Test				
A. modus ponens	1-10	2,4,9,10	2,4,9,10	2,4,9,10
B. modus tollens	11-20	14,15,17,18	12,14,18,20	18
C. denying the antecedent	21-30	23,28,29,30	22,23,28,30	30
D. affirming the consequent	31-40	31,32,35,39	31,32,35,39	35,39
Part II of Test				
E. conjunction	41-46	43,44,45	43,44,45	43,44,45
F. disjunction	47-52	47,51,52	47,51,52	51
G. converse, inverse, contrapos.	53-60	55,56,60	55,58,60	None
H. quantifiers:				
all	61-64	61,64	61,64	64
any	65-68	67,68	67,68	67,68
each	69-72	71	71	71
every	73-76	74	74	74
some	77-80	79,80	79,80	79
Total Numbers of Items	80	33	33	18

any questions, and (5) comment on the test as a whole. Items were grouped for the jury according to the type of logic which they were to test (Table 1); the purpose of each group of items was noted for the judges. Space was provided for their comments. Jurors were told that any items which were not marked would be considered valid items and might be used in the resulting test.

All of the jurors returned their comments and criticisms. Almost all of the questions were criticized, either positively or negatively, by at least one of the judges. Language of the test was considered appropriate. Mrs. Ford, the fourth grade teacher on the jury, wrote that "the vocabulary used in this test is on fourth grade level. No child should have any trouble vocabulary-wise." Several other judges agreed with this statement; none disagreed. Some of the judges commented on the words included in specific items, the need of extremely clear directions, and/or the need for "none of these" to be included as a choice on all items.

On the basis of the comments of jury members, a test was constructed for the pilot study. Hereafter, this new test will be called the pilot test (Appendix 2). Table 1 contains a list (Column 2) of the items chosen for the pilot test from the jury test. Questions 28, 29, 31, and 35 of the jury test were left unchanged in writing the pilot test; in each of the other questions used for the pilot test, one of the choices was changed to "none of these." In some questions, one or more words were changed to make the language simpler or to make the item not contrary to reality.

By distribution of different types of questions throughout the test, it was hoped to diminish the possibility of learning from one question to the next while taking the test. The first four items on the pilot test included one item of group A (questions 1-10 of the jury test), one item from group B (questions 11-20 of the jury test), one item from group C (questions 21-30 of the jury test), and one item from group D (questions 31-40 of the jury test). For example, items 1-4 of the pilot test were the same as items 14, 23, 31, and 4, respectively, of the jury test. The second, third, and fourth groups of four items of the first part of the pilot test were chosen similarly. Questions 41-57 of the pilot test were chosen from questions 41-80 of the jury test. Items of different types were also mixed in sequence of appearance in this, the second part of the pilot test.

Eight of the first sixteen items on the pilot test had a correct answer of "none of these," the fifth choice. These eight questions were distributed throughout Part I of the test. Other questions were assigned answers of 1, 2, 3, 4, or 5 by using a table of random numbers.

Answer sheets for the final test were to be machine-scored in such a way as to get subtotals of correct scores for the first sixteen items and for the next seventeen items. To obtain subtotals, a group of unmarked spaces had to be left between answers for the two parts of the test. Because of the arrangement of the answer sheet, it was decided to number the first group of questions 1-16 and the second group of questions, 41-57. The questions on the pilot test were also

numbered in this way in order to check the clarity of directions for answering questions in corresponding spaces on answer sheets.

Pilot Study

The pilot test of thirty-three items was then given to a twelfth-grade mathematics class of high ability students. It was also given to a fourth-grade class characterized by the teacher as being slow. It was assumed that the two classes represented the extremes of abilities to be tested. Directions for the test were improved after each testing in the pilot study. Scores are given in Table 2.

If a group of people took the pilot test by marking their answers at random, the means and standard deviations of their scores would be as in Table 3.

If students had relied entirely on guessing, the mean score for the first sixteen items would have been 3.20 and the distribution of scores would have been approximately normal with a standard deviation of 1.60. Note that all twelfth-grade students (Table 2) scored seven or more correct. Such a score is 2.37 σ above the mean of 3.20 and could have been achieved only 0.89% of the time through guessing alone.

Similarly, note that all of the twelfth graders had nine or more correct responses on the second part of the test; such a score could be achieved by chance less than 0.13% of the time. All of the twelfth graders had a score of at least sixteen on the entire test; such a score could be achieved by chance less than 0.13% of the time.

TABLE 2

NUMBERS OF CORRECT ANSWERS ON BOTH PARTS OF TEST
AS WELL AS ON ENTIRE TEST FOR EACH STUDENT
IN PILOT TEST CLASSES

Twelfth Grade			Fourth Grade		
Items 1-16	Items 41-57	All Items	Items 1-16	Items 41-57	All Items
16	14	30	9	9	18
14	15	29	12	5	17
12	17	29	7	10	17
13	15	28	7	8	15
12	13	25	6	8	14
11	14	25	7	7	14
10	13	23	7	7	14
13	9	22	6	7	13
12	10	22	5	7	12
9	13	22	6	5	11
9	13	22	6	5	11
11	10	21	3	8	11
9	12	21	6	4	10
8	13	21	6	4	10
8	12	20	2	8	10
8	9	17	5	5	10
7	9	16	2	8	10
			7	2	9
			3	5	8
			5	3	8
			6	2	8
			5	2	7
			3	4	7
			3	3	6
			5	0	5
Means 10.7	12.3	23.1	Means 5.6	5.4	11.0

Therefore, both parts of this test, as well as the entire test, are statistically valid for the twelfth-grade students in the pilot testing; it is very improbable that students scored as highly as they did by chance alone.

TABLE 3

MEANS AND FREQUENCIES FOR RANDOM ANSWER CHOICES ON PART I, PART II, AND THE ENTIRE TEST

	Mean	Standard Deviation
Part I	3.20	1.60
Part II	3.40	1.65
Total Test	6.60	2.98

Of the fourth-grade students in the pilot class, 76% (i.e., 19 students of 25) had five or more correct on the first part of the test; such a score could be achieved by chance only 4.01% of the time. Of the fourth graders, 64% (i.e., 16 students) had five or more correct on the second part of the test; such a score could be achieved by chance only 16.85% of the time. Of the fourth graders, 68% (i.e., 17 students) had ten or more correct on the entire test; such a score could be achieved by chance only 12.71% of the time. Both parts of this test as well as the entire test are probably statistically valid for most of the fourth graders in the pilot study; it is unlikely that students scored as highly as they did by chance alone. The first part of the test appears statistically more valid than either the second part of the test or the entire test.

After an item analysis was done on the scores from the pilot study, items 6, 11, 13, and 49 were eliminated from the pilot test because of a low index of discrimination; these items corresponded to items 29, 15, 17, and 56, respectively, of the jury test. Items 22, 20, 12, and 58, respectively, of the jury test were used to replace them. Changes were made to items 12, 20, and 58 as suggested by comments of jury members; no change was made in item 22.

Choices for some questions were rearranged in the hope that some choices might seem more appealing to those taking the test. Questions 1 and 4 of the pilot test were interchanged so that the first item on the test might be easier than it was on the pilot test; in pilot testing, eleven students correctly answered item 1, whereas thirty-four students correctly answered item 4.

Reliability of the pilot test was checked by the split-halves method. An odd-even breakdown of answers was not used since different types of items of the first part of the test were grouped by fours. Instead, the number of items answered correctly from items 1-4, 9-12, 41-44, and 49-52 was considered as one half, and the number of items answered correctly for the rest of the test, the other half of scores. A Pearson product-moment coefficient of correlation was computed from the two-way frequency distribution which resulted and gave the resultant coefficient of reliability of 0.93.

Reading Level

The reading level of the resulting test (Appendix 3) was checked by the Spache test. Eleven samples of about one hundred words each were noted along with the number of words not on the Stone list of unfamiliar words, which is used in the Spache test [50:131-132]. Data appear in Table 4. Average grade placement for the test was 2.1. This means "that a child with that level of ability could read the [material] with adequate comprehension and a reasonable number of oral reading errors" [50:127].

Computer Analysis

A more comprehensive analysis of test items was done after all testing had been done than had been done in the pilot study. An Iterative Bidimensional Item Analysis program was used to find and eliminate test items which (1) were answered correctly by more than 90% of respondents, (2) were answered correctly by less than 10% of the respondents, or (3) discriminated poorly when a biserial index of 0.30 was used.

None of the items was found to be too easy. Only item 43 was found to be too difficult; just 6.4% of the respondents correctly answered the item. Fourteen items, however, were rejected because of poor discrimination; Table 5 lists the item numbers along with their biserial index of discrimination. The large number of items rejected

TABLE 4

READING LEVEL OF TEST INSTRUMENT AS EVALUATED BY SPACHE TEST

Sample Number	No. of Words	No. of Sentences	No. of Words Not on Stone Word List	Ave. Sentence Length	% Hard Words	Estimated Grade Placement
1	102	21	6	4.86	6	2.0
2	100	19	2	5.26	2	1.8
3	101	23	6	4.39	6	2.0
4	100	18	4	5.55	4	2.0
5	101	20	4	5.05	4	2.0
6	100	21	6	4.76	6	2.0
7	102	21	9	4.86	9	2.3
8	102	17	6	6.00	6	2.2
9	100	17	6	5.88	6	2.2
10	100	17	7	5.88	7	2.2
11	100	17	6	5.88	6	2.2

TABLE 5

ITEMS REJECTED BECAUSE OF POOR DISCRIMINATION

Item Number	Biserial Index	Item Number	Biserial Index
2	0.059	15	0.092
3	0.174	47	0.285
4	0.286	48	0.074
6	0.047	49	0.216
7	0.126	52	0.283
11	0.046	53	0.244
13	0.268	55	0.145

because of poor discrimination was surprising because the discrimination of items had been checked with scores from the pilot groups. Four of the rejected items (i.e., 6, 11, 13, and 49), however, were items which had been substituted for items in the pilot test after analysis of the pilot scores. Perhaps more items had poor discrimination at this time than on the check on data from pilot groups because (1) total scores from the pilot group were, in general, either high or low, and (2) the index of discrimination was not the same on both checks of item discrimination. The pilot group consisted of only a slow fourth-and a fast twelfth-grade class, whereas the actual group tested for this study included classes like these, but also classes from each grade between the two. Scores varied accordingly; in the actual testing for this study, there were not only high and low scores, but also a large number of scores close to the mean of all scores. Hence, the pilot group consisted mainly of students who found the test items to be easy or difficult, not of students of all ability levels. Also, there were three items on the pilot test which would not have been acceptable under a biserial index of 0.30; for each of items 43, 52, and 53, only three more of the top 27% of answer sheets (i.e., eleven scores) had the item correct than did the bottom 27%. This would give items 52 and 53 biserial indices of 0.27; the indices given in Table 5 for these items are 0.283 and 0.244, respectively. Item 43 had been rejected because it was too difficult.

Table 6 lists the numbers of the eighteen acceptable items along with the percentage of respondents who correctly answered the

item, the item variance, and the biserial index of discrimination. Only these eighteen acceptable items were used in further test and item analyses. For later analyses, Part I of the test now consisted only of items 1, 5, 8, 9, 10, 12, 14, and 16; Part II of the test consisted only of items 41, 42, 44, 45, 46, 50, 51, 54, 56, and 57. Part I dealt with implications. Items 1, 8, 10, and 14 related to modus ponens and item 5 related to modus tollens, whereas item 9 had to do with errors resulting from affirming the consequent, and items 12 and 16 with denying the antecedent. Part II now dealt with conjunction (i.e., items 45, 51, and 57), disjunction (i.e., item 41), and quantifiers (i.e., items 42, 44, 46, 50, 54, and 56). There were no acceptable items to check students' understandings of converses, inverses, and contrapositives of statements.

TABLE 6

ACCEPTABLE ITEMS

Item Number	Percent Correct	Variance	Biserial Index
1	60.6	0.238	0.382
5	59.6	0.248	0.323
8	78.6	0.196	0.461
9	14.8	0.127	0.385
10	72.3	0.197	0.367
12	22.3	0.173	0.312
14	83.0	0.141	0.367
16	24.3	0.184	0.452
41	35.6	0.229	0.430
42	89.7	0.091	0.422
44	42.0	0.242	0.446
45	53.2	0.249	0.531
46	47.1	0.249	0.513
50	28.6	0.204	0.682
51	58.2	0.243	0.547
54	24.2	0.183	0.470
56	31.8	0.216	0.497
57	72.0	0.198	0.548

The second program run through the computer to judge the test was the Gulliksen's Item Analysis Program. The program computes the total test reliability and validity coefficients of both parts of the test, as well as giving item variances. The coefficient of total test reliability was found to be 0.76. This is considerably lower than the reliability of 0.93 of the pilot test scores. The discrepancy is very likely due to the fact that students in pilot groups were only at the extremes of age levels of the actual test sample. Students who found test items easy or difficult--not students of all abilities--were included in the pilot groups.

To determine test validity, scores on parts of the test were compared with scores on the total test. The coefficient of total test validity for Part I was found to be 0.80, and that for Part II, 0.92. This means that scores on Part I of the test are highly related to scores on the total test, and scores on Part II of the test are very highly related to scores on the total test.

An ideal item variance would be 0.25. Notice (Table 6) that items 1, 5, 41, 44, 45, 46, 50, 51, and 56 all have variances above 0.20. Only item 42 has a low variance.

Summary

An initial test of eighty items was written and judged for validity by a panel of eleven qualified experts. Comments of jury members were considered in modifying the initial test to obtain a pilot test of thirty-three items, which was administered to a

high-ability twelfth-grade class and a "slow" fourth-grade class. Directions to the test were found to be clear. Reliability of the pilot test was 0.93. The test was shown to be statistically valid for twelfth-grade students and probably statistically valid for fourth-grade students. An item analysis was done on scores from the testing done in the pilot study; four items from the pilot test were changed to arrive at the resulting two-part, thirty-three item multiple-choice test. Reading level of the final test instrument was found to be 2.1. Computer analyses further checked test reliability and validity and eliminated items which were too easy or had a low index of discrimination. In these ways, the resulting test instrument was shown to be valid.

CHAPTER IV

DATA COLLECTION AND TREATMENT

Data Collection

A county school system offering a desirable cross section of population was chosen for this study. In 1968, the system included ninety-nine schools; there were 2,873 teachers in the system, and on a typical day, 73,150 students were in attendance in the schools.

A letter briefly describing this proposed study (Appendix 4) was sent to the selected school system (1) asking for permission to use the schools in the study and (2) requesting an interview if one should be considered desirable. A resulting interview was held and permission was granted to use the system. School administrators required that the names of students not be taken from the county; adequate means of guaranteeing the anonymity of each student was decided upon.

Meetings with principals, assistant principals, and/or counselors at assigned schools (Appendix 5) resulted in choices of specific classes, days, and times for testing in the schools. School authorities had been asked for classes which included students of both sexes with a wide range of I.Q.'s. Forty-six classes were used in the study; they are described in Table 7.

TABLE 7

NUMBERS OF CLASSES IN THE VARIOUS GRADES AND SOCIOECONOMIC LEVELS USED IN THE STUDY

		Grade								
		4	5	6	7	8	9	10	11	12
SOCIO-ECONOMIC STATUS	High	2	2	2	2	1	2	2	2	1
	Medium	2	2	2	2	2	2	2	2	2
	Low	2	2	2	1	1	1	1	1	1

In each class, the same directions (Appendix 6) were given to students taking the test. After a brief description of the study and the general purpose and type of test, each student was given a test, an answer sheet, a data card, and a pencil. He was asked to put his name and the number on his answer sheet onto the data card. Then the directions on his test were read aloud as he read them to himself. Detailed answers were given to three examples, one each of three different types of questions. The answer to one of the examples was the fifth choice, "none of these"; students were thus given a case of using "none of these" choice in answering a question before they took the test. The hypothetical nature of test questions was stressed; students were asked several times to make believe, pretend, and suppose that the first sentence or sentences in each question were true. They were told that there was no time limit to the test and that papers would be collected when everyone had finished.

While students were taking the test, the researcher twice circulated around the test room to collect data cards and to check marks on answer sheets. Teachers in classes of grades four, five, and six were asked the names of their students who could not read at grade level 2.1, the reading level of the test; answer sheets and data cards of these students were later discarded. This was a power test and sufficient time was given for all students to complete the test; testing time varied from thirty-five to fifty minutes. School cumulative records were examined to obtain the sex, birth date, father's occupation, and most recent Kuhlman-Anderson I.Q. score for each student.

The four variables of sex (S), age (A), socioeconomic status (E), and intellectual ability (I) were all thought to affect students' understandings of logic. Two sex groupings (i.e., male, S = 1; and female, S = 2), three socioeconomic groupings (i.e., low, E = 1; medium, E = 2; and high, E = 3), three intelligence groups (i.e., I.Q. 89 and less, I = 1; I.Q. 91-109, I = 2; and I.Q. 111+, I = 3), and three age groups (i.e., 10 years 3 months, or younger, A = 1; 10 years 7 months to 13 years 3 months, A = 2; and 13 years 7 months, and older, A = 3) were used. Age groups had been divided into a small number of categories to eliminate the existence of many small or empty cells. Age levels were grouped in accordance with hypothesis 6 of Chapter I (page 7), which stated that there will be more variability within the scores of those of age eleven through thirteen than within the scores of those younger than eleven or the scores of those older than thirteen. Age group 2 was thus chosen to consist of students of ages from 10 years

7 months to 13 years 3 months. The other two age groups were then chosen accordingly. Gaps were left between the age groups to eliminate scores of students who were near the borderline between age groups; scores of children of ages 10 years 4 months to 10 years 6 months and 13 years 4 months to 13 years 6 months were discarded. Scores of children of I.Q. of 90 and of 110 were discarded for the same reason.

A total of 1279 students was tested; 860 of their scores were useable. Scores of 419 of the students were not included in the final data analysis because (1) the students could not read at grade level 2.1, (2) a Kuhlman-Anderson I.Q. score, birth date, and/or father's occupation was not given in their school records, or (3) their answer sheet was marked in a way indicating an invalid score.

Data Treatment

Up to this time, each answer sheet contained only answers to each of the thirty-three questions on the test. Now each answer sheet was given an identifying code number and was coded to designate the respondent's sex, age, socioeconomic status, and I.Q. level. Ages were recorded on the basis of students' age in years and months as of March 1, 1969, since testing had been done between February 15, and March 10, 1969. A code for socioeconomic status was assigned on the basis of a student's father's occupation according to the scheme presented by the Otis Dudley Duncan scale [38]. Students whose father's occupation ranked in the lower 30% of the Duncan scale were coded 1; those whose father's occupation ranked in the middle 40% of the scale,

coded 2; and those whose father's occupation ranked in the upper 30% of the scale, coded 3.

A system of Applied Multiple Linear Regression known as PERSUB* was used six times to obtain necessary F ratios of individual variables. Multiple-range tests [24] for significant variables were done by hand, as were (1) determinations of the variability of age groups on each part of the test and on the whole test, (2) F tests for significant differences between the variances of age groups on each part of the test and on the entire test, (3) computation of the means of various groups, and (4) an analysis of covariance of cell means of Part I of the test with respect to the cell means of Part II of the test.

*Robert A. Bottenberg and Joe H. Ward, Jr. Applied Multiple Linear Regression. Springfield: Clearinghouse for Federal Scientific and Technical Information, 1963.

CHAPTER V

DATA ANALYSIS

The following statistical hypotheses will be considered in this chapter:

1. There will be no statistically significant difference between means of correct scores for different age levels on Part I of the test.
2. There will be no statistically significant difference between means of correct scores for different age levels on Part II of the test.
3. There will be no statistically significant difference between means of correct scores for different age levels on the whole test.
4. There will be no statistically significant difference between means of correct scores for various ability levels on Part I of the test.
5. There will be no statistically significant difference between means of correct scores for various ability levels on Part II of the test.
6. There will be no statistically significant difference between means of correct scores for various ability levels on the whole test.
7. There will be no statistically significant difference between means of correct scores for different socioeconomic levels on Part I of the test.
8. There will be no statistically significant difference between means of correct scores for different socioeconomic levels on Part II of the test.

9. There will be no statistically significant difference between means of correct scores for different socioeconomic levels on the whole test.
10. There will be no statistically significant difference between means of scores of the two sexes on Part I of the test.
11. There will be no statistically significant difference between means of correct scores of the two sexes on Part II of the test.
12. There will be no statistically significant difference between means of correct scores of the two sexes on the whole test.
13. There will be no statistically significant interaction between the significant variables of ability group and age, socioeconomic level, or sex on Part I of the test.
14. There will be no statistically significant interaction between the significant variables of ability group and age, socioeconomic level, or sex on Part II of the test.
15. There will be no statistically significant interaction between the significant variables of ability group and age, socioeconomic level, or sex on the whole test.
16. There will be no statistically significant interaction between the significant variables of sex and age or socioeconomic group on Part I of the test.
17. There will be no statistically significant interaction between the significant variables of sex and age or socioeconomic group on Part II of the test.

18. There will be no statistically significant interaction between the significant variables of sex and age or socioeconomic group on the entire test.
19. There will be no statistically significant interaction between the significant variables of age and socioeconomic group on Part I of the test.
20. There will be no statistically significant interaction between the significant variables of age and socioeconomic group on Part II of the test.
21. There will be no statistically significant interaction between the significant variables of age and socioeconomic group on the entire test.
22. There will be no statistically significant higher order interaction between the significant variables of ability, age, or socioeconomic level or sexes on Part I of the test.
23. There will be no statistically significant higher order interaction between the significant variables of ability, age, or socioeconomic levels or sexes on Part II of the test.
24. There will be no statistically significant higher order interaction between the significant variables of ability, age, or socioeconomic levels or sexes on the entire test.
25. There will be no statistically significant difference between the variances of correct scores of age levels on Part I of the test.
26. There will be no statistically significant difference between the variances of correct scores of age levels on Part II of the test.

27. There will be no statistically significant difference between the variances of correct scores of age levels on the whole test.
28. There will be no statistically significant difference between means of correct scores associated with the two types of logic for any group.

The 0.05 level of significance was chosen for use throughout this study.

Examination of the Data

Table 8 shows the frequencies and mean scores of each of the fifty-four possible combinations of sex (two-value), age (three-value), socioeconomic level (three-value), and intelligence level (three-value).

TABLE 8

CELL MEANS, FREQUENCIES, AND DESCRIPTION

	Mean Scores				Cell Description			
Cell No.	Part I	Part II	Total Test	Frequency	Sex	Age	S-E Status	Intelligence
1	3.00	5.00	8.00	1	1	1	1	1
2	3.00	3.00	6.00	1	1	1	2	1
3	-	-	-	0	1	1	3	1
4	2.50	3.75	6.25	4	1	1	1	2
5	2.50	3.50	6.00	4	1	1	2	2
6	2.80	3.60	6.40	5	1	1	3	2
7	1.00	3.50	4.50	2	1	1	1	3
8	3.00	4.21	7.21	14	1	1	2	3
9	3.67	4.47	8.13*	15	1	1	3	3
10	2.93	2.47	5.40	15	1	2	1	1
11	2.96	3.29	6.14*	7	1	2	2	1
12	2.67	2.67	5.33*	3	1	2	3	1
13	3.15	3.54	6.69	13	1	2	1	2
14	3.23	3.70	6.93	40	1	2	2	2
15	3.63	4.23	7.87*	30	1	2	3	2
16	4.00	3.00	7.00	1	1	2	1	3
17	3.83	3.88	7.72*	18	1	2	2	3
18	4.11	5.04	9.14*	28	1	2	3	3
19	3.29	4.10	7.38*	21	1	3	1	1
20	3.64	3.36	7.00	11	1	3	2	1
21	3.00	4.20	7.20	10	1	3	3	1
22	4.52	5.30	9.82	23	1	3	1	2
23	5.05	6.15	11.20	41	1	3	2	2
24	4.63	5.98	10.60*	43	1	3	3	2
25	5.22	5.11	10.33	9	1	3	1	3
26	5.50	7.80	13.30	20	1	3	2	3
27	5.47	7.72	13.19	32	1	3	3	3

TABLE 8 (Continued)

Cell No.	Mean Scores: Part I	Part II	Total Test	Frequency	Cell Description: Sex	Age	S-E Status	Intelligence
28	2.50	1.00	3.50	2	2	1	1	1
29	2.00	1.00	3.00	1	2	1	2	1
30	-	-	-	0	2	1	3	1
31	2.33	2.83	5.17*	6	2	1	1	2
32	2.67	2.33	5.00	9	2	1	2	2
33	3.57	2.57	6.14	7	2	1	3	2
34	3.14	3.86	7.00	7	2	1	1	3
35	3.20	3.20	6.40	15	2	1	2	3
36	3.27	4.60	7.87	15	2	1	3	3
37	2.55	3.88	6.44*	9	2	2	1	1
38	2.64	2.18	4.82	11	2	2	2	1
39	4.67	4.67	9.33*	3	2	2	3	1
40	3.38	3.33	6.71	21	2	2	1	2
41	3.07	3.41	6.49*	41	2	2	2	2
42	3.80	3.88	7.68	25	2	2	3	2
43	3.33	3.67	7.00	3	2	2	1	3
44	4.24	4.47	8.71	17	2	2	2	3
45	4.45	4.21	8.66	29	2	2	3	3
46	3.26	3.79	7.05	19	2	3	1	1
47	3.62	3.69	7.31	13	2	3	2	1
48	4.40	6.20	10.60	5	2	3	3	1
49	3.93	4.74	8.67	43	2	3	1	2
50	4.34	5.37	9.71	35	2	3	2	2
51	4.60	5.00	9.60	30	2	3	3	2
52	5.00	5.83	10.83	12	2	3	1	3
53	5.62	6.69	12.31	26	2	3	2	3
54	5.55	7.53	13.09*	45	2	3	3	3
Total	4.03	4.77	8.79*	860				

*Total test mean is not equal to sum of means of the two parts of the test due to rounding off of numbers.

Hypotheses 1-3 deal with the significance of age with respect to children's understandings of deductive logic. Values of F ratios for Parts I and II of the test and for the total test were all greater than those necessary for statistical significance at the 0.05 level, as can be seen in Table 9.

TABLE 9

F RATIOS AND SIGNIFICANCE LEVELS OF AGE GROUPS ON BOTH PARTS OF TEST AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part I	6.9700	$P < 0.01$
Part II	8.4728	$P < 0.01$
Total Test	11.0953	$P < 0.01$

The null hypotheses state that there is no difference in the age groups' understandings of logic. If this is true, the probabilities of getting differences for the age groups as large as the differences obtained are less than 0.01. The means of each age group on each part of the test and on the entire test are shown in Table 10 and also graphically in Figure 1. Multiple-range tests indicated that for Part I of the test and for the total test, the mean of each age group was significantly different from the means of each of the other age groups at the 95% level of confidence. On Part II of the test, however, the means of age groups 1 and 3 and the means of age groups 2 and 3 were significantly different, while the means of age groups 1 and 2 were

not significantly different at the 95% level of confidence as indicated by a horizontal line under the 3.62 and 3.79 in Table 10. Hypotheses 1-3 were rejected. In other words, age made a difference in scores.

TABLE 10

MEANS AND FREQUENCIES OF AGE GROUPS

	Age Groups			Total
	A = 1 (to 11)	A = 2 (11-13)	A = 3 (13 on)	
Part I	3.04	3.53	4.63	--
Part II	3.62	3.79	5.75	--
Total Test	6.66	7.32	10.38	--
Frequency	108	314	438	860

Although age was already shown to be a significant factor in performance on the logic test, a more detailed analysis of means of age groups was desired. Mean scores were, therefore, determined for each age for both parts of the test and for the total test; they are listed in Table 11 and shown graphically in Figure 2. Because there were relatively few students under nine years of age or over seventeen years of age in this study, they were grouped with the nine- and seventeen-year-old students, respectively.

After age nine, there was an apparent steady increase in the mean scores on both parts of the test and on the entire test. The darkened lines in the graph of Figure 2 indicate year-intervals in

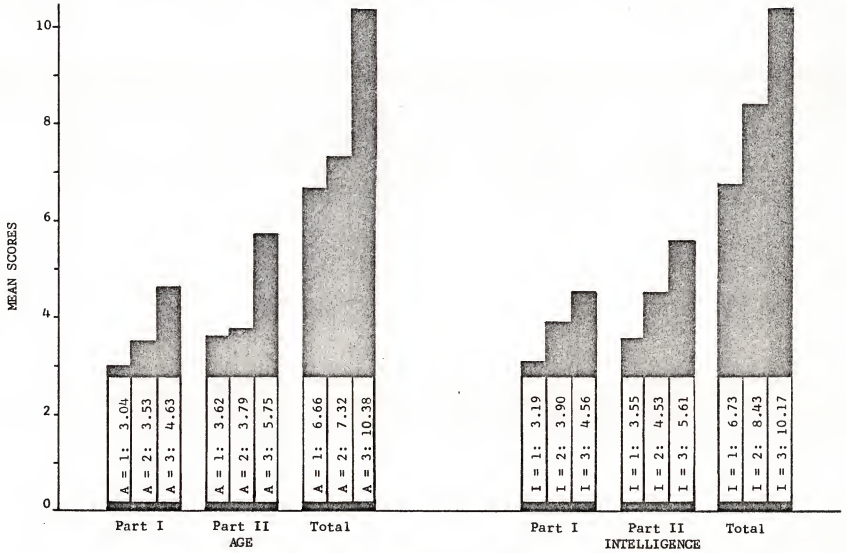

Figure 1. Mean Scores for Each Age Group on Part I, Part II, and Entire Test

which there were statistically significant differences (0.05 level) in mean scores. It can be seen that a statistically significant gain in mean scores on both parts of the test and on the total test begins at about age fourteen.

TABLE 11

MEANS AND FREQUENCIES OF EACH AGE ON BOTH PARTS OF TEST AND TOTAL TEST

Age	Means of Scores			Frequency
	Part I	Part II	Total Test	
9	3.09	3.73	6.82	89
10	2.92	3.49	6.41	88
11	3.55	3.75	7.29	119
12	3.83	3.85	7.68	105
13	3.95	4.47	8.42	62
14	4.14	4.81	8.95	81
15	4.49	5.76	10.25	91
16	4.91	6.07	10.98	114
17	5.03	6.56	11.59	111
Total	--	--	--	860

The mean scores of nine-year-old students was greater than the mean scores of ten-year-old students. An explanation for this might be the result of another study.

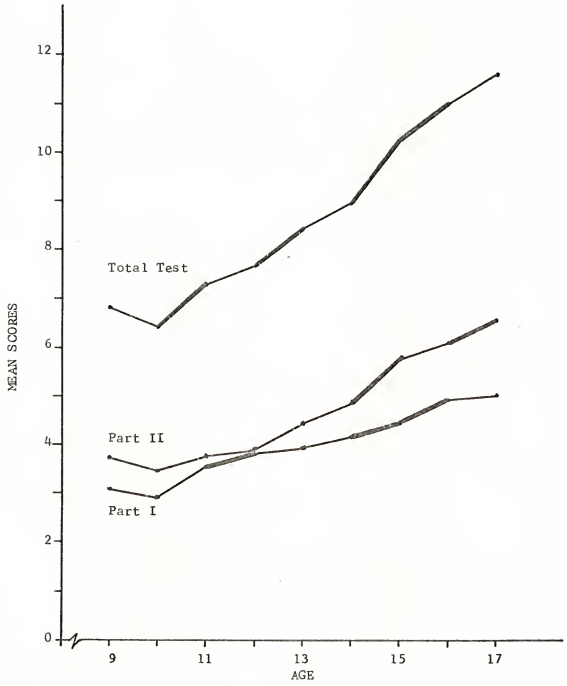

Figure 2. Mean Scores for Each Age Group on Part I, Part II, and Entire Test

Hypotheses 4-6 deal with the significance of intelligence with respect to children's understandings of deductive logic. Values of F ratios for Parts I and II of the test and for the total test were all greater than those necessary for statistical significance at the 0.05 level. See Table 12.

TABLE 12

F RATIOS AND SIGNIFICANCE LEVELS OF INTELLIGENCE GROUPS ON BOTH PARTS OF TEST AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part I	3.5712	$P < 0.01$
Part II	4.3224	$P < 0.01$
Total Test	5.5474	$P < 0.01$

The null hypotheses state that there is no difference in the intelligence groups' understandings of logic. If this is true, the probabilities of getting differences for the intelligence groups as large as the differences obtained are less than 0.01. The means of each intelligence group on each part of the test and on the entire test are shown in Table 13 and also graphically in Figure 1. Multiple-range tests indicated that for Part I, Part II, and the Total Test, the mean of each intelligence group was significantly different from the means of each of the other intelligence groups at the 95% level of confidence. Hypotheses 4-6 were rejected. In other words, intelligence made a difference in scores.

TABLE 13

MEANS AND FREQUENCIES OF INTELLIGENCE GROUPS

	Intelligence Groups			Total
	I = 1 (to 89)	I = 2 (91-109)	I = 3 (111+)	
Part I	3.19	3.90	4.56	--
Part II	3.55	4.53	5.61	--
Total Test	6.73	8.43	10.17	--
Frequency	132	420	308	860

Hypotheses 7-9 deal with the significance of socioeconomic level with respect to children's understandings of deductive logic. Socioeconomic level made a significant difference at the 95% level of confidence in scores on Part II of the test and on the Total Test, but not on Part I of the test. Values of F ratios for Part II of the test and for the Total Test were all greater than those necessary for statistical significance at the 0.05 level. The value of the F ratio for Part I of the test was less than that necessary for significance at this level. See Table 14.

TABLE 14

F RATIOS AND SIGNIFICANCE LEVELS OF SOCIOECONOMIC GROUPS ON BOTH PARTS OF TEST AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part I	1.0405	P = 0.41
Part II	1.5074	P = 0.03
Total Test	1.5315	P = 0.03

The null hypotheses state that there is no difference in the socioeconomic groups' understandings of logic. If this is true, the probabilities of getting differences for the age groups as large as the differences obtained are 0.41, 0.03, and 0.03 on the two parts of the test and the entire test, respectively. Hence, socioeconomic status was significant only on Part II and on the total test at the 95% level of confidence. The means of each socioeconomic group on each part of the test and on the total test are shown in Table 15, and also

TABLE 15

MEANS AND FREQUENCIES OF SOCIOECONOMIC GROUPS

	Socioeconomic Groups			
	E = 1 (low)	E = 2 (medium)	E = 3 (high)	Total
Part I	3.60	3.94	4.39	--
Part II	4.13	4.57	5.58	--
Total Test	7.73	8.51	9.77	--
Frequency	211	324	325	860

graphically in Figure 3. Multiple-range tests on Part II and the total test indicated that for both of them, the mean of each socioeconomic group was significantly different from the means of each of the other socioeconomic groups at the 95% level of confidence. Hypothesis 7 was accepted, but hypotheses 8 and 9 were rejected. Socioeconomic status sometimes made a difference in scores.

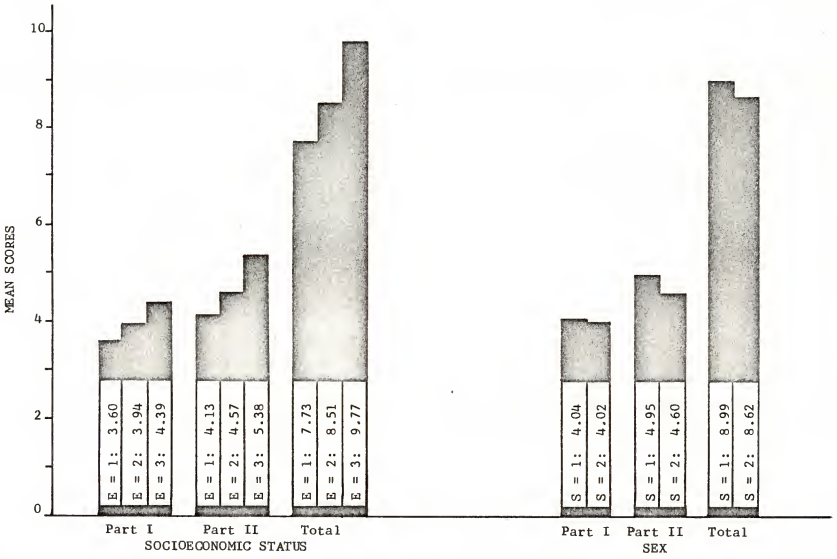

Figure 3. Mean Scores for Each Age on Both Parts of Test and on Entire Test

Hypotheses 10-12 deal with the significance of sex with respect to children's understandings of logic. Values of F ratios for both parts of the test and for the total test were all less than those necessary for significance. See Table 16.

TABLE 16

F RATIOS AND SIGNIFICANCE LEVELS OF SEX GROUPS ON BOTH PARTS OF TEST AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part I	0.8394	P =0.71
Part II	1.2096	P =0.22
Total Test	1.0966	P =0.34

The null hypotheses state that there is no difference in the sex groups' understandings of logic. If this is true, the probabilities of getting differences for the age groups as large as the differences obtained are 0.71, 0.22, and 0.34 on the two parts of the test and the entire test, respectively. Hence, sex was not significant on either part of the test or on the entire test at the 95% level of confidence. The means of each sex group on each part of the test and on the total test are shown in Table 17 and also graphically in Figure 3. When there was no statistically significant difference between a pair of means, the pair was underlined in the appropriate table.

TABLE 17

MEANS AND FREQUENCIES OF SEX GROUPS

	Sex Groups		
	S = 1 (male)	S = 2 (female)	Total
Part I	4.04	4.02	--
Part II	4.95	4.60	--
Total Test	8.99	8.62	--
Frequency	411	449	860

Hypotheses 10-12 were accepted. Sex did not make a difference in scores.

More F tests were computed to determine whether various interactions of significant variables made a significant difference in test scores. F tests were done on A x I, A x E, E x I, and A x E x I groupings on Part II and on the total test; an F test was done on A x I interaction groups for Part I of the test. Since sex was not significant anywhere in the test instrument, no F tests were done on interaction groups involving sex; for the same reason, no F tests were done on interaction groups involving socioeconomic groups on Part I of the test.

The interaction of age and intelligence did make a significant difference in scores. Values of F ratios for Part I, Part II, and the total test were all greater than those necessary for statistical significance at the 0.05 level as can be seen in Table 18. The null hypotheses state that there is no difference in the A x I interaction

TABLE 18

F RATIOS AND SIGNIFICANCE LEVELS OF AxI INTERACTION GROUPS ON BOTH PARTS OF TEST AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part I	39.0443	$P < 0.01$
Part II	44.5291	$P < 0.01$
Total Test	61.2198	$P < 0.01$

groups' understandings of logic. If this is true, the probabilities of getting differences for the A x I interaction groups as large as the difference obtained are less than 0.01. Table 19 lists the means of each A x I interaction group on each part of the test and on the total test. Results of multiple-range tests are shown by the horizontal line or lines under cell means in Table 19. Whenever the means of two interaction groups are underlined by the same horizontal line, the interaction groups were not significantly different at the 95% level of confidence; whenever two interaction groups are not joined by the same horizontal line, they were significantly different at the level of confidence. For example, in Part II of Table 20, interaction groups (11) and (12), (21) and (22), (31) and (13), and (21 and (13) were not significantly different, whereas (33) and (11), (12) and (13), (22) and (23), and (31) and (32) were significantly different. Here (ab) represents the interaction group in which A = a and I = b.

Extreme differences in cell size affected multiple-range tests in some instances. For example, in Part II of Table 20, (11) and (13),

TABLE 19

MEANS AND FREQUENCIES OF A x I INTERACTION GROUPS

Part I									
Cell Number	(11)	(12)	(21)	(13)	(22)	(31)	(23)	(32)	(33)
Frequency	5	35	48	68	170	79	96	215	144
Mean	2.60	2.77	2.88	3.21	3.36	3.42	4.16	4.51	5.47
Part II									
Cell Number	(11)	(12)	(21)	(22)	(31)	(13)	(23)	(32)	(33)
Frequency	5	35	48	170	79	68	96	215	144
Mean	2.20	2.94	2.94	3.69	4.00	4.07	4.38	5.46	7.16
Total Test									
Cell Number	(11)	(12)	(21)	(22)	(13)	(31)	(23)	(32)	(33)
Frequency	5	35	48	170	68	79	96	215	144
Mean	4.80	5.71	5.81	7.05	7.28	7.42	8.53	9.96	12.13

Note: (ab) represents the cell in which A = a and I = b.

(11) and (23), (12) and (13), and (31) and (23) were not significantly different, but (21) and (13), and (31) and (32) were significantly different at the 95% level of confidence. The discrepancy of two interaction groups being significantly different (ex., (11) and (31): difference of means, 0.82), while two interaction groups with a smaller difference in means (ex., (21) and (13): difference of means, 0.41), were significantly different, was probably caused by the fact that groups had small frequencies (ex., the frequency of (11) was just 5).

Similarly, for Part I, (11) and (23), and (11) and (32) were not significantly different, but (12) and (23), (12) and (32), (21) and (23), (13) and (23), and (22) and (23) were significantly different at the 95% level of confidence. There were no such discrepancies for scores on the total test in A x I interaction groups.

A further analysis of means of A x I interaction groups was done. Mean total-test scores were determined for each age for each intelligence group; they are listed in Table 20 and are also shown graphically in Figure 4. In Figure 4, the darkened lines in the graph indicate year-intervals in which there was a statistically significant difference (0.05 level) in mean scores. The dotted lines in the graph point out ages at which there were statistically significant differences (0.05 level) in means between the indicated intelligence groups.

Several important conclusions can be drawn from Table 20 and Figure 4. The students in the upper-intelligence group attained a higher level of understanding of deductive logic than did those in the middle group, and those in the middle group a higher level of understanding than those in the lower group. There were three year-intervals

of statistically significant growth for students in the upper-intelligence group; there were two for students in the middle-intelligence group; and there was one for students in the lower-intelligence group. The differences in mean scores of the three intelligence groups were all statistically significant (0.05 level) after age fourteen.

TABLE 20

MEANS AND FREQUENCIES FOR EACH AGE FOR EACH INTELLIGENCE GROUP

	Intelligence Groups					
	I = 1 (89 and lower)		I = 2 (91-109)		I = 3 (111+)	
Age	Mean	Frequency	Mean	Frequency	Mean	Frequency
9	4.50	4	6.24	25	7.34	60
10	6.75	4	5.64	48	7.39	36
11	5.15	13	6.88	68	8.82	38
12	6.00	25	7.83	52	9.24	28
13	6.70	10	8.20	35	9.88	17
14	5.86	14	8.34	44	12.39	23
15	7.30	23	10.43	35	12.16	33
16	6.70	20	10.73	52	13.19	42
17	9.26	19	11.11	61	13.65	31
Totals		132		420		308

An examination of Figure 4 will reveal remarkably similar "growth" patterns for the two upper-intelligence groups. There was a period of slow increase in understanding of logic followed by a brief period (i.e., one year) of pronounced growth; this period of rapid growth was followed by another period of slow growth. Children in the

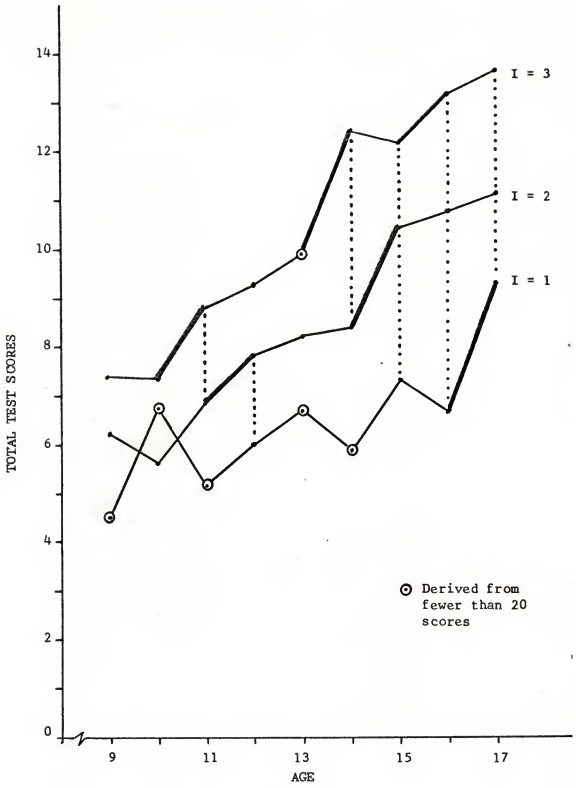

Figure 4. Mean Scores for Each Intelligence Group for Each Age on the Total Test

upper-ability group made a considerable gain in understanding of logic between age thirteen and age fourteen. Children in the middle-ability group, however, made a considerable gain a year later in their lives--between ages fourteen and fifteen. Children in the low-ability group made a considerable gain between the ages sixteen and seventeen. It appears that students showed the greatest improvement in understanding of deductive logic at a mental age between fourteen and fifteen. The size of the low-ability group was small for some ages; consequently, the mean scores may not be representative for the group. The t tests showed that the means of the two upper intelligence groups were significantly different (0.05 level) at almost every age.

The interaction of age and socioeconomic status did make a significant difference in scores. Values of F ratios for Part II of the test and for the total test were each greater than those necessary for statistical significance at the 0.05 level, as can be seen in Table 21. The null hypotheses state that there is no difference in the A x E interaction groups' understandings of logic. If this were true, the probabilities of getting differences for the A x E interaction groups as large as the differences obtained are less than 0.01.

TABLE 21

F RATIOS AND SIGNIFICANCE LEVELS OF A x E INTERACTION GROUPS ON PART II AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part II	30.8518	$P < 0.01$
Total Test	40.1811	$P < 0.01$

Table 22 lists the means of each A x E interaction group on Part II and on the total test. Again, results of multiple-range tests are shown by the horizontal lines below means in Table 22. Recall that two interaction groups were not significantly different if and only if their means were underlined by the same horizontal line. Exceptions are (11) and (23), and (11) and (31) of Part II of Table 22; neither pair of interaction groups was significantly different. Here (ab) represents the interaction group in which A = a and E = b.

A further analysis of means of A x E interaction groups was done. Mean total-test scores were determined for each age for each socioeconomic group; they are listed in Table 23 and are also shown graphically in Figure 5. In Figure 5, the darkened lines in the graph indicate year-intervals in which there was a statistically significant difference (0.05 level) in mean scores. The dotted lines in the graph point out ages at which there were statistically significant differences (0.05) in means between the indicated socioeconomic groups.

Several important conclusions can be drawn from Table 23 and Figure 5. Those students in the upper socioeconomic group attained a higher level of understanding of deductive logic than did those in the middle group, and those in the middle group a higher level of understanding than those in the lower group. Each socioeconomic group showed growth in their understandings of deductive logic in almost every year-interval shown. There were two year-intervals of statistically significant "growth" for students in the upper socioeconomic group; there was one for students in the middle socioeconomic group; and there was none for students in the lower socioeconomic group. The period of

TABLE 22

MEANS AND FREQUENCIES OF A x E INTERACTION GROUPS

Part II									
Cell Number	(21)	(11)	(12)	(22)	(13)	(23)	(31)	(32)	(33)
Frequency	62	22	44	134	42	118	127	146	165
Mean	3.21	3.32	3.32	3.59	4.10	4.31	4.72	4.85	6.46
Total Test									
Cell Number	(11)	(12)	(31)	(22)	(13)	(23)	(31)	(32)	(33)
Frequency	22	44	62	134	42	118	127	146	165
Mean	5.86	6.25	6.32	6.91	7.50	8.30	8.75	10.66	11.39

Note: (ab) represents the cell in which A = a and E = b.

greatest increase in understandings of logic occurred earlier for students in higher socioeconomic groups. The period of greatest "growth" occurred between ages fourteen and fifteen for the high socioeconomic group, between the ages thirteen and fourteen for the middle socioeconomic group, and between the ages twelve and thirteen for the low socioeconomic group.

TABLE 23

MEANS AND FREQUENCIES FOR EACH AGE FOR EACH SOCIOECONOMIC GROUP

	Socioeconomic Groups					
	E = 1 (low)		E = 2 (medium)		E = 3 (high)	
Age	Mean	Frequency	Mean	Frequency	Mean	Frequency
9	5.95	20	6.41	34	7.71	35
10	6.08	11	5.93	41	7.03	36
11	5.94	18	6.84	56	8.40	45
12	6.59	27	7.43	37	8.83	41
13	7.24	21	7.88	24	10.65	17
14	7.68	34	9.73	22	10.00	25
15	9.00	26	9.91	32	11.33	33
16	9.30	27	11.46	37	11.50	50
17	9.70	27	11.80	41	12.65	43
Totals		211		324		325

The middle socioeconomic group showed the greatest overall gain in scores from ages nine to seventeen. At ages nine, ten, eleven, twelve, and thirteen, there were statistically significant differences between the means of the middle and upper socioeconomic

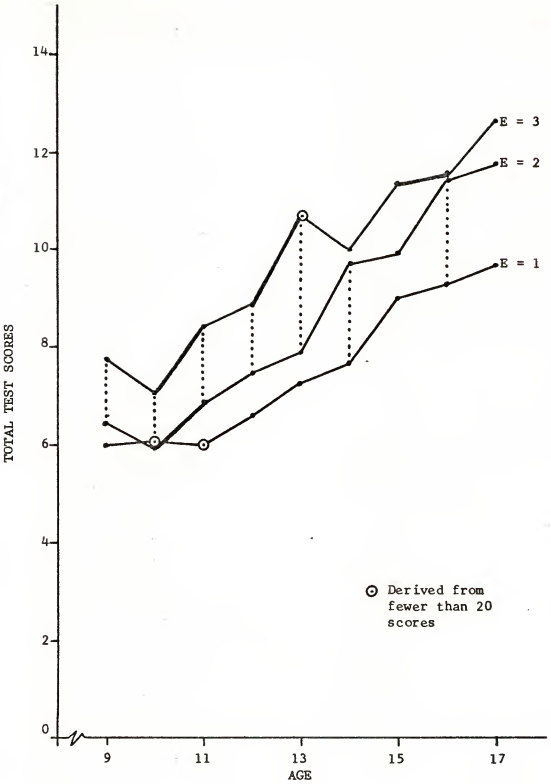

Figure 5. Mean Scores for Each Socioeconomic Group for Each Age on the Total Test

groups; there was also relatively little difference between the means of the middle and low socioeconomic groups. At age fourteen and sixteen, however, there was a statistically significant difference between the means of the middle and lower socioeconomic groups; there was then relatively little difference between the means of the middle and upper socioeconomic groups.

The interaction of socioeconomic groups and intelligence groups also made a significant difference in scores. Values of F ratios for Part II and the total test were each greater than those necessary for statistical significance at the 0.05 level as can be seen in Table 24.

TABLE 24

F RATIOS AND SIGNIFICANCE LEVELS OF E x I INTERACTION GROUPS ON PART II AND ON THE TOTAL TEST

	F Ratio	Probability that F ratio Is Not Significant
Part II	12.5128	$P < 0.01$
Total Test	15.7279	$P < 0.01$

The null hypotheses state that there is no difference in the A x E interaction groups' understandings of logic. If this is true, the probabilities of getting differences for the age groups as large as the differences obtained are less than 0.01. Table 25 lists the means of each E x I Interaction group on Part II and the total test. Results of multiple-range tests are shown again by the horizontal lines in the table. In addition to interaction groups whose means are indicated to

TABLE 25

MEANS AND FREQUENCIES OF E x I INTERACTION GROUPS

Part II									
Cell Numbers	(21)	(11)	(12)	(22)	(31)	(13)	(32)	(23)	(33)
Frequency	44	67	110	170	21	34	140	110	164
Mean	3.09	3.54	4.31	4.49	4.52	4.73	4.76	5.29	6.02

Total Test									
Cell Number	(21)	(11)	(12)	(31)	(22)	(32)	(13)	(23)	(33)
Frequency	44	67	110	21	170	140	34	110	164
Mean	6.79	6.61	8.03	8.05	8.30	8.91	9.00	9.72	10.72

Note: (ab) represents the cell in which E = a and I = b).

be not significantly different, groups (11) and (31), (21) and (31), and (31) and (12) of Part II and groups (21) and (31) of the total test were not significant at the 95% level of confidence. Here (ab) represents the interaction group where E = a and I = b.

The interaction of age, intelligence, and socioeconomic groups also made a difference in scores. Values of F ratios for Part II and for the total test were each greater than those necessary for statistical significance at the 0.05 level, as can be seen in Table 26. The null hypotheses state that there is no difference in the A x E x I groups' understandings of logic. If this is true, the probabilities of getting differences for the age groups as large as the differences obtained are less than 0.01.

TABLE 26

F RATIOS AND SIGNIFICANCE LEVELS OF A x E x I INTERACTION GROUPS ON PART II AND ON TOTAL TEST

	F Ratio	Probability that F Ratio Is Not Significant
Part II	15.5083	$P < 0.01$
Total Test	20.9915	$P < 0.01$

Multiple-range tests were not done because the large numbers of cells and small sizes of many of them could result in conclusions of only questionable validity. Hypotheses 22-24 were rejected. A x E x I interaction groups did make a difference in scores.

Hypotheses 25-28 deal with the variability within the scores of age groups. Variances of each age group for each part of the test and for the total test were determined. They are given in Table 27 along with a frequency distribution of the scores of each age group on each part of the test and on the entire test. F ratios for each part of the test and for the entire test are given in Table 28. The figures indicate that on Part I of the test the variance of age group 3 was statistically significantly greater than the variances of age groups 1 and 2 at the 0.05 level. There was no statistically significant difference between the variances of age groups 1 and 2, as is indicated by the line under their variances in Table 28. There was not a statistically significant difference between the variance of any pair of age groups on Part II of the test. On the entire test, the variance of age group 3 was significantly (0.05 level) greater than the variance of age group 2, which in turn was significantly greater than the variance of age group 1. Hypothesis 26 was accepted, but hypotheses 25 and 27 were rejected. Age made a difference in the variance of children's understandings of deductive logic only in Part I of the test and in the total test.

A final analysis was done on the fifty-two nonempty cells to determine the coefficient of correlation between the means of cells on Part I of the test and the means of cells on Part II of the test. The Pearson product-moment coefficient of correlation was found to be just 0.51. There was only a medium-sized correlation between the means of cells of Part I of the test and the means of cells on Part II of the test.

TABLE 27

FREQUENCIES AND VARIANCES OF AGE GROUPS ON BOTH PARTS OF TEST AND ON ENTIRE TEST

	Age Groups								
	A = 1			A = 2			A = 3		
Score	Part I	Part II	Total Test	Part I	Part II	Total Test	Part I	Part II	Total Test
18			0			3			2
17			0			0			18
16			0			1			17
15			0			2			23
14			1			4			34
13			1			3			42
12			1			4			41
11			4			16			35
10		0	7		4	24		27	47
9		1	5		5	36		48	36
8	0	4	14	3	10	48	17	56	41
7	1	1	19	3	6	51	44	50	29
6	2	9	24	5	28	46	49	45	30
5	13	19	12	78	39	20	134	61	24
4	23	20	6	80	67	35	91	56	11
3	31	21	10	48	79	15	63	45	5
2	22	20	4	66	45	6	30	36	2
1	15	11	0	27	25	0	5	14	1
0	1	2	0	4	6	0	5	0	0
Totals	108	108	108	314	314	314	438	438	438
Variance	1.851	3.421	5.923	2.240	3.825	6.680	2.591	6.250	12.889

TABLE 28

F RATIOS BETWEEN VARIANCES OF DIFFERENT AGE GROUPS ON EACH PART OF THE TEST AND ON THE ENTIRE TEST

	F Ratio Between A3 and A2	F Ratio Between A2 and A1	F Ratio Between A3 and A1	Significant F Ratio at 0.05 Level
Part I	1.16	1.63	1.93	1.20
Part II	1.21	1.12	1.13	1.30
Total Test	1.40	1.83	2.18	1.28

Summary

Data were analyzed mainly by means of twenty-nine F tests and necessary multiple-range tests. Twelve F tests dealt with the significance of age, socioeconomic, sex, and intelligence groups on each part of the test and on the total test. Additional F tests were done on interactions of significant variables. Analyses were done on specific ages and A x I groups after it was found that age and the A x I interaction were statistically significant. Significance was found according to Table 29. Eight other F tests checked to find significant differences in variances of age groups. Finally, a coefficient of correlation compared the means of scores of respondents in each cell on the two parts of the test. Tables and graphs were used to describe the results of all tests.

TABLE 29

SIGNIFICANCE OF F TESTS ON EACH PART OF TEST
AND ON TOTAL TEST
(5% Level)

	Part I	Part II	Total Test
Sex	No	No	No
Socioeconomic	No	Yes	Yes
Intelligence	Yes	Yes	Yes
Age	Yes	Yes	Yes
A x I	Yes	Yes	Yes
A x E	---	Yes	Yes
E x I	---	Yes	Yes
A x E x I	---	Yes	Yes

CHAPTER VI

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

Summary

Although much has been written about logic and mathematics in past decades, interest in the subject and its relationship to mathematics has increased tremendously in the past decade. Fourteen studies which dealt with the relationship between the two disciplines were examined; four of them were closely related to this study.

Shirley Hill found that six- through eight-year-old children were able to recognize valid inferences. William Miller concluded that eighth-, tenth-, and twelfth-grade students were able to recognize valid, but could not recognize invalid, inferences. Jean Piaget found that the ability of children to think logically develops rapidly from about age eleven to about age fourteen. Robert Ennis concluded that basic principles of logic were usually not mastered by age eleven or twelve.

This study was similar to that by Ennis. Both dealt with the same variables in dealing with over 800 students in grades four through twelve. There were several differences in the studies, however. No students in this study were given any instruction in logic. This study included students of high, medium, and low intellectual ability, whereas Ennis's study involved only high-ability students. A thirty-

three item multiple-choice test was used in this study rather than a seventy-two item yes-no-maybe test.

After the literature had been reviewed to determine what aspects of logic were considered relevant to the study of logic, eighty multiple-choice questions were written to test understandings of deductive logic. The questions were submitted to a jury of experts for their judgement of the validity of the items; as a result, forty-seven of the items were rejected. The remaining questions were given to a pilot group of a slow fourth-grade class and a high-ability twelfth-grade class as a two-part test. Part I of the test checked students' understandings of implications and Part II, their understandings of conjunction, disjunction, and quantifiers. An item analysis was done along with checks of reliablity and statistical validity. As a result of these checks, four items were replaced and several others were modified. The resulting test was administered to 1279 students from forty-six fourth- through twelfth-grade classes in a county school system; the students came from a wide range of socioeconomic backgrounds and intellectual abilities in both sexes for each grade. Scores of 860 of the students were used in this study. After all of them had been tested, statistical checks eliminated from further analysis fourteen of the thirty-three test items because of poor discrimination and one because of its difficulty.

A program of Applied Multiple Linear Regression was used in obtaining F ratios for the scores on the logic test and its two parts to determine whether sex, age, socioeconomic status, or intelligence were statistically significant. The 0.05 level of statistical significance was used throughout this study. More F ratios were computed to

determine if the interactions of significant variables were also significant. When values of F ratios were found to be significant, multiple-range tests were applied to the appropriate means to test the significance of differences between the various mean scores. The variability of age groups was checked to determine which age group, if any, had significantly greater variability. For the nonempty cells, mean scores on both parts of the test were obtained and compared to find the coefficient of correlation between the two parts of the test.

Conclusions

Conclusions of the study are the following:

1. Age made a statistically significant difference in the ability of students to understand logic. Oldest students did best, the students in the middle age group did the next best, and students in the youngest age group did the least well on Part I of the test--which checked their understandings of implications--and on the whole test. On Part II of the test--which checked understandings of conjunction, disjunction, and quantifiers--the oldest students did significantly better than did each of the other groups; there was no significant difference between the two younger age groups on this part of the test.
2. Intelligence made a statistically significant difference in the ability of students to understand logic. In this study, students of I.Q. 111+ did significantly better than did students of I.Q. 91-109, who in turn did significantly better than did students of I.Q. 89 or less on both parts of the test and on the entire test.

3. Socioeconomic status did not make a statistically significant difference in students' understandings of implications, but did make a statistically significant difference in their understandings of conjunction, disjunction, and quantifiers, and in their scores on the total test. Students in the high socioeconomic group did significantly better than did students of medium socioeconomic level, who in turn did significantly better than students of low socioeconomic status on the test of logic and on the second part of the test.

4. There was no statistically significant difference between the means of scores of the two sexes on either part of the test or on the entire test.

5. F tests of interaction groups indicated that the interaction of age and intelligence was a significant factor in total test scores and in scores on both parts of the test. For the two upper-intelligence groups, there was a period of slow increase in understanding of logic followed by a brief period of pronounced growth; this period of rapid growth was followed by another period of slow growth. For each age group, the period of most rapid growth in understandings of logic occurred between the mental ages of fourteen and fifteen.

6. The interaction of age and socioeconomic status made a statistically significant difference in total test scores and in scores on Part II of the test.

7. The interaction of intelligence and socioeconomic status made a statistically significant difference in total test scores and in scores on Part II of the test.
8. The interaction of age, intelligence, and socioeconomic status made a statistically significant difference in total test scores and in scores on Part II of the test.
9. There was a statistically significant difference in the variance of correct scores between age groups on Part I of the test. Scores of students in the age group of older students had significantly greater variance than those of students in the middle age group, where the variance was in turn significantly greater than that of the younger students. On Part II of the test and on the total test, the variability of the scores of students in the age group of older students was significantly greater than the variability of the scores of students in the two other groups.
10. There was a medium-sized (i.e., 0.51 coefficient of correlation) relationship between the cell means of correct scores on Part I of the test and the cell means of the correct scores on Part II of the test.

The conclusions of this study agree with some of the conclusions of past research, but disagree with others. This study agrees with that by Ennis in that age, intelligence, and socioeconomic status were found to be significant with respect to students' understandings of logic, but that sex was not significant in this respect. It also agrees with Ennis's work in showing that the basic principles of logic are not generally mastered by age eleven or twelve. This study disagrees with

that by Ennis, however, with respect to stages in the development of understandings of logic. Ennis "did not find evidence for the existence of stages in development of knowledge of logic among students not receiving special training"; he found "a fairly gradual overall improvement in knowledge of logic from age ten to eighteen among the students" which he studied [12:26-27]. This study found a period of slow increase in understanding of logic followed by a brief period of rapid growth between mental ages fourteen and fifteen, and then followed by another period of slow increase in understandings of logic.

Piaget concluded that development of understandings of logic proceeds rapidly from around eleven or twelve years to around fourteen or fifteen years of age. This study found a rapid development over a period of just one year. Piaget, however, referred to chronological ages and this study, to mental ages in this respect. According to the data of this study, the period of rapid growth may occur at any chronological age from twelve to sixteen, depending upon the intellectual ability of the child.

Although Hill dealt with first, second, and third grade children only, she also concluded that sex differences were not significant. Miller also found that students in grades eight, ten, and twelve understand some types of deductive logic.

Limitations

Some of the limitations of this study are the following: (1) all students may not have had the same amount and type of informal instruction in logic, (2) I.Q. test scores were several years old in some instances, (3) students in different socioeconomic groups may have different patterns of reasoning, (4) all students were not tested at the same time of day or under the exact same conditions, and (5) specific words, phrases, or questions may have been difficult for some students.

Implications

Many implications for education can be made from this study. A few of them are the following:

1. This study does not support the use of sex as a factor to be considered in planning instruction in deductive logic.
2. The factors of age, socioeconomic status, and intelligence should be considered in planning instruction in deductive logic.
3. Planning for instruction in logic should consider the need for variability in approaches for students. This need seems to be greater in the case of the older students.
4. Students who have received no planned instruction in logic, it would appear, will experience difficulty in the deductive proof aspects of mathematics prior to the mental age of about fourteen or fifteen years. It may be wise to emphasize intuitive approaches to mathematics for students who have not reached these ages. It must be pointed out, however, that no evidence is available to

show whether the period of improvement in understandings of logic can be hastened by means of planned instruction.

5. In general, brighter students can be taught more difficult subject matter which is directly related to understandings of deductive logic than can those relatively less bright.

6. Some questions have been raised by this study. Some of them are the following:

Will a replication of this study with larger samples and a more extensive test corroborate the pattern of development of understandings of deductive logic shown in this study?

Will a longitudinal study of students corroborate the pattern of development of understandings of logic? Such a study might begin with students of mental age around twelve and test them each six months over a three-year period.

Can the period of most rapid improvement in understandings of deductive logic be brought about at an earlier age by planned instruction?

What experiences can be provided to students of low socioeconomic status to improve their understandings of deductive logic? Does the home background fail to provide adequate experiences in reasoning? Will planned instruction which provides suitable experiences such as decision-making produce improvement in ability in deductive logic?

How do socioeconomic levels affect understandings of logic for each I.Q. group?

What aspects of current curricula and instruction aid the development of students' understandings of deductive logic?

What is the relative effectiveness of various strategies in teaching logic to various groups of students? Such strategies may include Venn diagrams, truth tables, symbols (such as $p \rightarrow q$, $p \vee q$, and $\sim p$), and laboratory situations.

APPENDIX 1

JURY TEST

Box 12963 University Station
Gainesville, Florida 32601
January 2, 1969

Dear ,

About three weeks ago, you received a letter asking you if you would be willing to check the validity of proposed questions for a logic test. You indicated your willingness to assist in this task. These are the proposed questions. There are more items of each type than will be needed. The test will be given to samples of all fourth through twelfth grade students in Orlando. Please read each question; then

(1) Circle ambiguous words, phrases, or questions.

(2) Write the words "too easy" and/or "too hard" next to questions which are too easy for twelfth graders and/or too hard for fourth graders, respectively.

(3) Put an X in front of any question which does not test what it is supposed to test or which tests more than it is supposed to test.

(4) Make any additional comments which you may want to make next to the questions in the spaces provided.

(5) Comment on the test as a whole in the space provided at the end of the test items.

Any items which are not marked will be considered valid items and may be used in the resulting test.

When this test is completed, the different types of questions will be cyclically arranged. A table of random numbers will be used to designate correct answers to consecutive questions as a, b, c, d, or e.

A stamped, addressed return envelope has been enclosed for your convenience. Thank you for your time and consideration. Your help and cooperation are greatly appreciated.

Sincerely,

Robert S. Matulis

DIRECTIONS: THIS IS NOT A TEST OF HOW MUCH YOU KNOW; IT IS A TEST OF HOW WELL YOU CAN THINK. IN EACH TEST ITEM, SENTENCES ARE GIVEN WHICH YOU ARE TO SUPPOSE TO BE TRUE. THE SENTENCES ARE FOLLOWED BY FIVE OTHER SENTENCES WHICH ARE MARKED a, b, c, d, AND e. ONE OF THE FIVE IS TRUE BECAUSE THE FIRST SENTENCES ARE SUPPOSED TO BE TRUE. ON YOUR ANSWER SHEET, MARK THE LETTER OF THE SENTENCE WHICH IS TRUE WHEN THE FIRST SENTENCES ARE TRUE. NOTICE THE FOLLOWING EXAMPLES:

Example 1: IF APPLES ARE PURPLE, THEN PEARS ARE BLUE. APPLES ARE PURPLE.
a) Pears are blue.
b) Apples are white.
c) Apples are orange.
d) Pears are purple.
e) None of these is true.

The correct answer is a.

Example 2: I AM A BOY OR I LIKE PEPPER.
a) I like pepper.
b) I like salt.
c) I am a boy.
d) I am a girl.
e) None of these is true.

Which is the correct answer?

- - - - - - - - - - - - - -

INSTRUCTION TO JURY: QUESTIONS 1-10 ARE SUPPOSED TO TEST STUDENTS' RECOGNITION OF THE VALIDITY OF PROOFS INVOLVING MODUS PONENS - i.e., If p, then q. p. Therefore q.

1. PAUL IS GOING IF JOHN IS COMING. JOHN IS COMING.
a) Paul is coming.
b) John is going.
c) John is not coming.
d) Paul is going.
e) Paul is a traveling salesman.

COMMENT:

2. IF JOHN IS HANDSOME, THEN I AM A MONKEY'S UNCLE. JOHN IS HANDSOME.
   a) I am handsome.
   b) I am ugly.
   c) I am a monkey's uncle.
   d) I am not a monkey's uncle.
   e) John is a monkey's uncle.

   COMMENT:

3. IF BOYS HAVE BLUE EYES, THEY HAVE CURLY HAIR. BOYS HAVE BLUE EYES.
   a) Boys do not have curly hair.
   b) Boys have curly eyes.
   c) Boys have blue hair.
   d) Boys do not have curly blue hairy eyes.
   e) Boys have curly hair.

   COMMENT:

4. I OWN A CAR IF I OWN A TEMPEST. I OWN A TEMPEST.
   a) I own a car.
   b) I do not own a car.
   c) I do not own a Tempest.
   d) I do not own a Camaro.
   d) I don't know how to drive.

   COMMENT:

5. JACK PLAYS WELL IF HE LIKES BASEBALL. JACK LIKES BASEBALL.
   a) Jack drops many fly balls.
   b) He can't pitch well.
   c) He can't throw well.
   d) Jack plays well.
   e) Jack is a slow runner.

   COMMENT:

6. MUD IS WET IF SEEDS GROW. SEEDS GROW.
   a) Mud grows.
   b) Seeds do not grow.
   c) Mud is dry.
   d) Mud is wet.
   e) Seeds are wet.

   COMMENT:

7. IF THIS BOX HAS NUTS IN IT, THIS CAN HAS FRUIT IN IT. THERE ARE NUTS IN THIS BOX.
   a) There is fruit in this can.
   b) There are nuts in this can.
   c) There is fruit in this box
   d) There are no nuts in this box.
   e) There is no fruit in this can.

   COMMENT:

Ronnie Not Sue the Sam Not Here

8. IF RATS EAT CATS, THEN DOGS EAT FROGS. RATS EAT CATS.
   a) Dogs eat rats.
   b) Dogs eat cats.
   c) Dogs eat frogs.
   d) Frogs eat cats.
   e) Cats eat frogs.

   COMMENT:

9. IF JIM READS BOOKS, HE WILL BE SMART. JIM READS BOOKS.
   a) Jim will be smart.
   b) Jim will not be smart.
   c) Jim will not read books.
   d) Smart people read books.
   e) Jim is dumb.

   COMMENT:

10. IF THE WIND BLOWS IN MAY, IT WILL BE FOGGY IN APRIL. THE WIND BLOWS IN MAY.
    a) It will be foggy in April.
    b) It will be foggy in May.
    c) It will not be foggy in April.
    d) It will not be foggy in May.
    e) There will be no wind in May.

    COMMENT:

INSTRUCTION TO JURY: QUESTIONS 11-20 ARE SUPPOSED TO TEST STUDENTS' RECOGNITION OF THE VALIDITY OF PROOFS INVOLVING MODUS TOLLENS - i.e., If p, then q. Not q. Therefore not p.

11. TOMMY IS STRONG IF TOMMY IS A GIRL. TOMMY IS NOT STRONG.
    a) Tommy is strong.
    b) Tommy is a girl.
    c) Tommy is a boy.
    d) Tommy is a weak girl.
    e) Tommy is a 90 pound weakling.

    COMMENT:

12. IF I DRIVE A CAR, THEN I NEED A DRIVER'S LICENSE. I DON'T NEED A DRIVER'S LICENSE.
    a) I don't drive a car.
    b) I need a driver's license.
    c) I drive a car.
    d) My car needs a driver's license.
    e) I don't know how to drive.

    COMMENT:

13. GREEN IS SALTY IF BLACK IS A SOUND.
    a) Black is not a sound.
    b) Black is a sound.
    c) Green is salty.
    d) Dark green is a sound.
    e) Sound is not salty.

    COMMENT:

14. IF JIM IS A BOY, THEN JIM IS GOOD LOOKING. JIM IS NOT GOOD LOOKING.
    a) Jim is not a boy.
    b) Jim is not a girl.
    c) Jim is not a man.
    d) Jim is not a woman.
    e) Jim is clean.

    COMMENT:

15. IF DAY ENDS, IT IS NOISY. IT IS NOT NOISY.
    a) Day does not end.
    b) Day ends.
    c) It is 3 A.M.
    d) It is midnight.
    e) It is noisy.

    COMMENT:

16. EAST IS SOUTH IF WEST IS NORTH EAST IS NOT SOUTH.
    a) West is South.
    b) East is North.
    c) West is not North.
    d) West is South.
    e) South is North.

    COMMENT:

17. IF IT IS GETTING COLD, THEN SPRING IS OVER. SPRING IS NOT OVER.
    a) Spring is over.
    b) It is getting cold.
    c) It is not getting cold.
    d) It is getting warm.
    e) It is summer.

    COMMENT:

18. THE CAR IS BLUE IF THE TRUCK IS GREEN. THE CAR IS NOT BLUE
    a) The car is blue.
    b) The car is green.
    c) The truck is green.
    d) The truck is blue.
    e) The truck is not green.

    COMMENT:

19. IF BIRDS HAVE FINS, THEN ORLANDO IS A STATE. ORLANDO IS NOT A STATE.
    a) Orlando is a state.
    b) Birds have fins.
    c) Birds do not have fins.
    d) There are no birds in Orlando.
    e) Fish have wings.

    COMMENT:

20. IF MR. JONES IS OLD, HE HAS GRAY HAIR. MR. JONES DOES NOT HAVE GRAY HAIR.
    a) Mr. Jones has brown hair.
    b) Mr. Jones is bald.
    c) Mr. Jones is young.
    d) Mr. Jones is old.
    e) Mr. Jones is not old.

    COMMENT:

INSTRUCTION TO JURY: QUESTIONS 21-30 ARE SUPPOSED TO TEST STUDENTS' RECOGNITION OF THE INVALIDITY OF PROOFS WHICH AFFIRM THE CONSEQUENT-- i.e., If p, then q. q. Therefore p.

Carl Carl

21. IF JIM WANTS TO PASS, HE WILL NOT STUDY. JIM WILL NOT STUDY.
    a) Jim wants to pass.
    b) Jim does not want to pass.
    c) Jim is a good student.
    d) Jim has nothing to do.
    e) None of these.

    COMMENT:

22. IF THE LIGHT IS GREEN, I'LL CROSS THE STREET. I'LL CROSS THE STREET.
    a) The light is green.
    b) The light is yellow.
    c) The light is red.
    d) The light is burned out.
    e) None of these.

    COMMENT:

23. RUTH IS MARRIED IF DALE IS A GIRL. RUTH IS MARRIED.
    a) Dale is a girl.
    b) Dale is a boy.
    c) Dale is married.
    d) Dale is not married.
    e) None of these.

    COMMENT:

24. IF YOU DON'T CALL ME, I'LL CALL YOU. I'LL CALL YOU.
    a) You called me.
    b) You didn't call me.
    c) You are calling me.
    d) The phone is busy.
    e) None of these.

    COMMENT:

25. IF A CAT IS A DOG, THEN CATS BARK. CATS BARK.
    a) Cats are dogs.
    b) Dogs are cats.
    c) Dogs do not bark.
    d) Cats bark.
    e) None of these.

    COMMENT:

26. YOU CAN GET LOST IF YOU WANT TO. YOU CAN GET LOST.
    a) You cannot get lost.
    b) You want to get lost.
    c) You are lost.
    d) You do not want to get lost.
    e) None of these.

    COMMENT:

27. FOG IS HARD IF GRASS IS PINK. FOG IS HARD.
    a) Grass is hard.
    b) Fog is hard.
    c) Fog is pink.
    d) Fog is soft.
    d) None of these.

    COMMENT:

28. IF BROWN COWS GIVE CHOCOLATE MILK, THEN WHITE COWS GIVE WHITE MILK. WHITE COWS GIVE WHITE MILK.
    a) Brown cows give chocolate milk.
    b) Brown cows give white milk.
    c) White cows do not give white milk.
    d) White cows give chocolate milk.
    e) None of these.

    COMMENT:

29. MIKE IS JIM'S SON IF DON IS JIM'S FATHER. MIKE IS JIM'S SON.
    a) Mike is Don's father.
    b) Mike is Don's son.
    c) Don is Jim's father.
    d) Don is Mike's father.
    e) None of these.

    COMMENT:

30. IF JANE IS BEAUTIFUL, MEN WILL LOOK AT HER. MEN LOOK AT JANE.
    a) Jane is beautiful
    b) Jane is ugly.
    c) Jane has on a mini-skirt.
    d) Jane is smart.
    e) None of these.

    COMMENT:

INSTRUCTION TO JURY: QUESTIONS 31-40 ARE SUPPOSED TO TEST STUDENTS' RECOGNITION OF THE INVALIDITY OF PROOFS WHICH DENY THE ANTECEDENT - i.e., If p, then q. Not p. Therefore not q.

31. IF THE CAR RUNS, IT HAS GAS. THE CAR DOES NOT RUN.
   a) The car does not have gas.
   b) The car is red.
   c) The car has a flat tire.
   d) Nobody started the car.
   e) None of these.

   COMMENT:

32. IF IRON IS STRONG, IT IS NOT A METAL. IRON IS NOT STRONG.
   a) Iron is strong.
   b) Iron is not a metal.
   c) Iron is not expensive.
   d) Iron is soft.
   e) None of these.

   COMMENT:

33. IF YOU ARE HONEST, THEN YOU STEAL. YOU ARE NOT HONEST.
   a) You steal.
   b) You do not steal.
   c) You are not honest.
   d) You stole a car.
   e) None of these.

   COMMENT:

34. IF GREEN IS MONKEY, THEN RED IS COW. GREEN IS NOT MONKEY.
   a) Red is cow.
   b) Red is not cow.
   c) Green is monkey.
   d) It is Christmas.
   e) None of these.

   COMMENT:

35. IF IT RAINS, I'LL BE THERE. IT DOES NOT RAIN.
   a) I won't be there.
   b) I'll get wet.
   c) I'll be there.
   d) I need an umbrella.
   e) None of these.

   COMMENT:

36. GOATS CAN FLY IF THEY HAVE WINGS. GOATS DO NOT HAVE WINGS
    a) Goats can fly.
    b) Goats cannot fly.
    c) Goats have fur.
    d) Goats can swim.
    e) None of these.

    COMMENT:

37. IF COWS ARE NOT PURPLE, THEN ELEPHANTS ARE NOT PINK. COWS ARE PURPLE.
    a) Elephants are pink.
    b) Elephants are not pink.
    c) Cows are purple.
    d) Cows are blue.
    e) None of these.

    COMMENTS:

38. LEAD MELTS IF IT IS COLD. LEAD IS NOT COLD.
    a) Lead melts.
    b) Lead freezes.
    c) Lead is warm.
    d) Lead does not melt.
    e) None of these.

    COMMENTS;

39. IF JOHN IS BUSY, SUE IS TOO. JOHN IS NOT BUSY.
    a) Sue is busy.
    b) Sue is not busy.
    c) Sue is too.
    d) John is busy.
    e) None of these.

    COMMENTS:

40. IF AIRPLANES CRASH, THEN TRAINS STOP. AIRPLANES DO NOT CRASH.
    a) Airplanes crash.
    b) Trains do not stop.
    c) Trains do not stop.
    d) Airplanes stop
    e) None of these.

    COMMENTS:

INSTRUCTION TO JURY: QUESTIONS 41-46 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF CONJUNCTIVE SENTENCES.

41. IT WILL RAIN AND SNOW.
    a) It will rain.
    b) It will not rain.
    c) It will not snow.
    d) It might snow.
    e) It might not rain.

    COMMENTS:

42. THIS DOG IS YOURS AND THIS DOG IS A MOTHER.
    a) This dog is your mother.
    b) This dog is not yours.
    c) This dog is not a mother.
    d) Your dog is a mother.
    e) Your mother is a dog.

    COMMENTS:

43. IT IS JULY AND SNOW IS FALLING.
    a) It is December.
    b) Snow never falls in January.
    c) It is snowing in July.
    d) It is not snowing.
    e) The sky is green with orange dots.

    COMMENTS:

44. CURT IS IN THE DIRT AND GUY IS IN THE SKY.
    a) Curt is in the sky.
    b) Guy is in the dirt.
    c) Curt is not in the dirt.
    d) Guy is not in the sky.
    e) Guy is in the sky.

    COMMENTS:

45. BIRDS HAVE HORNS AND COWS ARE PINK.
    a) Birds have pink horns.
    b) Cows have pink horns.
    c) Birds are pink.
    d) Cows have horns.
    e) Cows are pink.

    COMMENTS:

46. ROSES ARE RED AND VIOLETS ARE BLUE.
    a) If roses are red, then violets are not blue.
    b) If violets are not blue, then roses are not red.
    c) Roses are not red.
    d) Roses are not red and violets are not blue.
    e) Violets are blue.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 47-52 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF DISJUNCTIVE STATATEMENTS:

47. ART IS SMART OR MARY IS MERRY.
    a) Art is merry.
    b) Mary is smart.
    c) If Art is dumb, Mary is sad.
    d) If Mary is not merry, Art is smart.
    e) Art is smart; Mary is merry, too.

    COMMENTS:

48. I AM 300 YEARS OLD OR I AM 24 FEET TALL.
    a) I am 300 years old.
    b) I am 24 feet tall.
    c) If I am not 300 years old, I am not 24 feet tall.
    d) If I am not 24 feet tall, I am not 300 years old.
    e) If I am not 24 feet tall, I am 300 years old.

    COMMENTS:

49. JANE IS BEAUTIFUL OR SMART.
    a) Jane is beautiful, so she is smart.
    b) Since Jane is dumb, she is ugly.
    c) If Jane is not smart, she is beautiful.
    d) Jane is beautiful; she is smart, too.
    e) If Jane is not smart, she is not beautiful.

    COMMENTS:

50. SAM IS BIG OR SUE IS SHORT.
    a) Since Sam is small, Jane is tall.
    b) Sam is big; Jane is short.
    c) If Sam is short, Jane is not short.
    d) If Sam is big, Jane is small.
    d) If Jane is tall, Sam is big.

    COMMENTS:

51. MY GUN IS RUSTY OR BROKEN.
    a) My gun is rusty.
    b) My gun is broken.
    c) I have a rusty broken gun.
    d) If I have a gun that is not rusty, it is broken.
    e) I want a new gun!

    COMMENTS:

52. BOB KNOWS THAT 2 + 2 = 4 OR HE CAN SPELL "CAT."
    a) Bob knows that 2 + 2 = 4.
    b) Bob knows that C A T spells "cat."
    c) If Bob does not know that 2 + 2 = 4, then he knows that C A T spells "cat."
    d) Bob can multiply.
    e) If Bob can't spell "cat," he thinks 2 + 2 = 22.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 53-60 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF CONVERSE, INVERSE, AND CONTRAPOSITIVES OF STATEMENTS.

53. IF DAN IS A MAN, THEN MARY IS TIRED.
    a) If Mary is a man, then Dan is tired.
    b) Dan is not a man if Mary is not tired.
    c) Dan is not tired if Mary is not a man.
    d) If Dan is not a man, then Mary is not tired.
    e) If mary is not tired, then Dan is not a man.

    COMMENTS:

54. THERE ARE NO CLOUDS IN THE SKY IF IT IS RAINING.
    a) It is raining if there are clouds in the sky.
    b) If it is not raining, there are clouds in the sky.
    c) There are no clouds in the sky if it is not raining.
    d) If there are no clouds in the sky, it is raining.
    e) It is not raining if there are clouds in the sky.

    COMMENTS:

55. IF SAILORS SWIM, FISH FRY.
    a) If sailors do not swim, fish do not fry.
    b) If fish fry, sailors swim.
    c) If fish do not fry, sailors do not swim.
    d) Things that swim are not fish.
    e) Things that fry are not sailors.

    COMMENTS:

56. I WILL GO IF YOU STAY.
    a) I will stay if you go.
    b) You will go if I stay.
    c) You will not stay if I will not go.
    d) If I will go, you will stay.
    e) If you do not stay, I will not go.

    COMMENTS:

57. IF DRAGONS BREATHE FIRE, THEY BURN THEIR LIPS.
    a) If dragons do not breathe fire, they do not burn their lips.
    b) If dragons burn their lips, they breathe fire.
    c) If dragons do not burn their lips, they do not breathe fire.
    d) If dragons breathe fire, they do not burn their lips.
    e) Dragons who breathe fire also breathe smoke.

    COMMENTS:

58. IF MEN WEAR RED SUSPENDERS, THEY FIGHT FIRES.
    a) If men do not wear red suspenders, they do not fight fires.
    b) Men do not fight fires if they wear red suspenders.
    c) If men fight fires, they wear red suspenders.
    d) Men wear red suspenders if they do not fight fires.
    e) If men do not fight fires, they do not wear red suspenders.

    COMMENTS:

59. IF MEN DO NOT LIE, THEY ARE NOT GOOD MEN.
    a) Men who lie are good men.
    b) Good men lie.
    c) Bad men do not lie.
    d) If men are not good, they lie.
    e) Liars are not bad men.

    COMMENTS:

60. MARY IS NOT SICK IF SHE ATE A PURPLE APPLE.
    a) Mary is not sick.
    b) Mary ate a purple apple.
    c) If Mary is sick, she has not eaten a purple apple.
    d) If Mary is not sick, she ate a purple apple.
    e) If Mary has not eaten a purple apple, she is sick.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 61-64 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF STATEMENTS INVOLVING THE WORD "ALL."

61. JIM LOST ALL OF HIS MARBLES.
    a) Jim now has no marbles.
    b) Jim now has just one marble.
    c) Jim now has more than one marble.
    d) Jim had a dozen marbles before he lost them.
    e) Jim has three marbles.

    COMMENTS:

62. ALL MEN ARE CIRCLES.
    a) No men are circles.
    b) Mr. Jones is a man, so he is a circle.
    c) A circle is a man.
    d) Mr. Smith is a man. He is not a circle.
    e) Mrs. Johnson is not a man; she is not a circle.

    COMMENTS:

63. ALL BOYS HAVE RED HAIR. JACK, SAM, MARK, AND JIM ARE BOYS; JANE IS NOT A BOY.
    a) Jane has blond hair.
    b) Jack does not have brown hair.
    c) Sam has green hair.
    d) Mark's hair is black.
    e) Jim's hair is not red.

    COMMENTS:

64. ALL SAILORS SWIM. ART, BEN, CARL, AND DAN ARE SAILORS.
    a) Art can swim.
    b) Ben cannot swim.
    c) Carl does not know how to swim.
    d) Dan can swim better than Ben.
    e) Art cannot swim.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 65-68 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF STATEMENTS INVOLVING THE WORD "ANY."

65. ANYBODY WHO DRIVES A FORD IS RICH.
    a) Mr. Jones drives a 1969 Cadillac, so he is poor.
    b) Mr. Smith is rich, since he drives a Rolls Royce.
    c) Since Mr. Johnson drives a Ford, he is not rich.
    d) Miss Gridge rides a bicycle; she is very poor.
    e) Mrs. Sludge drives an old Ford, so she is rich.

    COMMENTS:

66. ANY WOOD IS WET.
    a) Glass is wet.
    b) Wooden baseball bats are wet.
    c) Wooden shoes are cool.
    d) Pencils are made of wood; they are not wet.
    e) Rain is not wet.

    COMMENTS:

67. ANN DOES NOT HAVE ANY MONEY.
    a) Ann has three dimes.
    b) Ann has two dollar bills.
    c) Ann has no nickels.
    d) Ann has at least 1¢.
    e) Ann is rich.

    COMMENTS:

68. ANY ADDITION PROBLEM IS EASY.
    a) 3 x 4 is an easy problem.
    b) 67 - 66 is a hard problem.
    c) 2 + 2 is a hard problem.
    d) 37,981 + 84,277 is an easy problem.
    e) 2 + 3 + 4 + 5 is a hard problem.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 69-72 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF STATEMENTS INVOLVING THE WORD "EACH."

69. MARK HAS TEN PENCILS. EACH ONE IS RED. HE IS WRITING WITH ONE OF THEM.
    a) It is yellow.
    b) It is green.
    c) It is blue.
    d) It is red.
    e) It has no eraser.

    COMMENTS:

70. EACH OF MY EYES IS BLUE.
    a) My right eye is blue.
    b) My right eye is green.
    c) My left eye is orange.
    d) My left eye is brown.
    d) I have two black eyes.

    COMMENTS:

71. I HAVE FIVE COINS. EACH OF THEM IS A PENNY.
    a) I have five pennies.
    b) I have just one penny
    c) I do not have 5¢.
    d) I have five dimes.
    e) I have no pennies.

    COMMENTS:

72. I HAVE AN ARITHMETIC BOOK, A SPELLING BOOK, A SCIENCE BOOK, AND AN ENGLISH BOOK. EACH OF MY BOOKS IS GREEN.
    a) My science book is green.
    b) My arithmetic book is yellow.
    c) The color of my spelling book is not green.
    d) The color of my arithmetic book is purple.
    e) My English book is light pink.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 73-76 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF STATEMENTS INVOLVING THE WORD "EVERY."

73. EVERY CAR HAS A FLAT TIRE.
    a) Fords have one flat tire.
    b) Chevrolets have no tires.
    c) Bicycles have two tires.
    d) Jim's hot rod has four tires.
    e) Mrs. Smith's car has no flat tires.

    COMMENTS:

74. EVERY TEACHER IS BEAUTIFUL.
    a) My teacher is beautiful.
    b) Every teacher in this school is ugly.
    c) Blond teachers are not beautiful.
    d) Arithmetic teachers are not beautiful.
    e) Short English teachers are ugly.

    COMMENTS:

75. EVERY PIECE OF METAL IS SHINY.
    a) Every shiny thing is metal.
    b) Metal cans are not shiny.
    c) Glass windows are not shiny.
    d) Paper cups are shiny.
    e) A pin is metal, so it is shiny.

    COMMENTS:

76. EVERY TREE HAS THREE BRANCHES.
    a) Palm trees have three branches.
    b) Elm trees do not have any branches.
    c) Oak trees have just one branch.
    d) Orange trees may have only two branches.
    e) Vines have no branches since they are not trees.

    COMMENTS:

INSTRUCTIONS TO JURY: QUESTIONS 77-80 ARE SUPPOSED TO TEST STUDENTS' UNDERSTANDINGS OF THE TRUTH VALUES OF STATEMENTS INVOLVING THE WORD "SOME."

77. SOME ARITHMETIC TEACHERS ARE BEAUTIFUL. MR. PLOP, MR. UGA, MISS ICK, AND MR. GLOP ARE ARITHMETIC TEACHERS.
    a) Mr. Plop is not beautiful.
    b) Mr. Uga is more beautiful than Miss Ick.
    c) Mr. Glop is beautiful.
    d) Miss Ick is not beautiful.
    e) At least one arithmetic teacher is beautiful.

    COMMENTS:

78. I LIKE SOME KINDS OF FOOD.
    a) I like candy.
    b) I like beer.
    c) There is at least one kind of food that I like.
    d) I hate candy.
    e) I like all food.

    COMMENTS:

79. SOME BOYS LIKE GIRLS.
    a) Jim likes Mary.
    b) Bill hates Jane.
    c) At least one boy likes girls.
    d) At least one girl likes boys.
    e) Tony does not like Carol.

    COMMENTS:

80. SOME GIRLS IN THIS ROOM ARE SHORT.
    a) No girls in this room are short.
    b) Some girls in this room are tall.
    c) Mary is short if she is in this room.
    d) Jane is in this room, so she is tall.
    e) There is at least one short girl in this room.

    COMMENTS:

COMMENTS ON THE TEST AS A WHOLE:

APPENDIX 2

PILOT TEST

DIRECTIONS: THIS IS NOT A TEST OF HOW MUCH YOU KNOW; IT IS A TEST OF HOW YOU THINK. IN EACH TEST ITEM, ONE OR TWO SENTENCES ARE GIVEN WHICH YOU ARE TO ACCEPT AS TRUE. THESE SENTENCES ARE FOLLOWED BY FIVE CHOICES WHICH ARE MARKED 1, 2, 3, 4, AND 5. ONE OF THE FIVE IS ALWAYS TRUE BECAUSE OF THE TRUTH OF THE FIRST SENTENCES. ON YOUR ANSWER SHEET, MARK THE LETTER OF THE CHOICE WHICH YOU ACCEPT AS TRUE. LOOK AT THE FOLLOWING EXAMPLES:

Example 1: I AM A BOY AND I LIKE PEPPER.
1 - I like salt.
2 - I like pepper.
3 - I hate sugar.
4 - I am a girl.
5 - None of these.

The best answer is "2."

Example 2: IF APPLES ARE PURPLE, THEN PEARS ARE BLUE.
1 - Apples are orange.
2 - Apples are white.
3 - Pears are blue.
4 - Pears are purple.
5 - None of these.

What is the best answer?

Example 3: IF MY PENCIL IS RED, IT HAS AN ERASER.
MY PENCIL HAS AN ERASER.
1 - My pencil is red.
2 - My pencil is not red.
3 - My pencil is yellow.
4 - My pencil is black with pink stripes.
5 - None of these.

PART I:

1. IF JIM IS A BOY, THEN JIM IS GOOD LOOKING. JIM IS NOT GOOD LOOKING.
   1 - Jim is not a girl.
   2 - Jim is not a boy.
   3 - Jim is not a woman.
   4 - Jim is not a man.
   5 - None of these

2. SUE IS MARRIED IF DALE IS A GIRL. SUE IS MARRIED.
   1 - Dale is a girl.
   2 - Dale is a body.
   3 - Dale is married.
   4 - Dale is not married.
   5 - None of these.

3. IF THE CAR RUNS, IT HAS GAS. THE CAR DOES NOT RUN.
   1 - The car does not have gas.
   2 - The car is red.
   3 - The car has a flat tire.
   4 - Nobody started the car.
   5 - None of these.

4. I OWN A RED CAR IF I OWN A FORD. I OWN A FORD.
   1 - I do not own a red car.
   2 - I own a red car.
   3 - I do not own a Ford.
   4 - I do not own a Dodge.
   5 - None of these.

5. THE CAR IS BLUE IF THE TRUCK IS GREEN. THE CAR IS NOT BLUE.
   1 - The car is blue.
   2 - The car is green.
   3 - The truck is green.
   4 - The truck is not green.
   5 - None of these.

6. MIKE IS JIM'S SON IF DON IS JIM'S FATHER. MIKE IS JIM'S SON.
   1 - Mike is Don's father.
   2 - Mike is Don's son.
   3 - Don is Jim's father.
   4 - Don is Mike's father.
   5 - None of these.

7. IF IRON IS STRONG, IT IS GRAY. IRON IS NOT STRONG.
   1 - Iron is strong.
   2 - Iron is gray.
   3 - Iron is not gray.
   4 - Iron is soft.
   5 - None of these.

8. IF JOE IS HANDSOME, THEN I AM A MONKEY'S UNCLE. JOE IS HANDSOME.
   1 - I am handsome.
   2 - Joe is a monkey's uncle.
   3 - I am not a monkey's uncle.
   4 - I am a monkey's uncle.
   5 - None of these.

9. IF JANE IS PRETTY, MEN WILL LOOK AT HER. MEN LOOK AT JANE.
   1 - Jane is pretty.
   2 - Jane is ugly.
   3 - Jane has on a mini-skirt.
   4 - Jane is smart.
   5 - None of these.

10. IF THE WIND BLOWS IN MAY, IT WILL BE FOGGY IN APRIL. THE WIND BLOWS IN MAY.
   1 - It will not be foggy in April.
   2 - It will not be foggy in May.
   3 - It will be foggy in April.
   4 - It will be foggy in May
   5 - None of these.

11. IF THE SKY IS CLOUDY, STUDENTS ARE NOISY. STUDENTS ARE NOT NOISY.
   1 - The sky is not cloudy.
   2 - The sky is cloudy.
   3 - It is raining.
   4 - Students are noisy.
   5 - None of these.

12. IF IT RAINS, I'LL BE THERE. IT DOES NOT RAIN.
   1 - I won't be there.
   2 - I'll get wet.
   3 - I'll be there.
   4 - I need an umbrella.
   5 - None of these.

13. IF IT IS GETTING WARM, THEN SPRING IS OVER. SPRING IS NOT OVER.
   1 - Spring is over.
   2 - It is getting warm.
   3 - It is summer.
   4 - It is not getting warm.
   5 - None of these.

14. IF JIM READS BOOKS, HE WILL BE SMART. JIM READS BOOKS.
   1 - Jim will be smart.
   2 - Jim will not be smart.
   3 - Jim will not read books.
   4 - Smart people read books.
   5 - None of these.

15. IF BROWN COWS GIVE CHOCOLATE MILK, THEN WHITE COWS GIVE WHITE MILK. WHITE COWS GIVE WHITE MILK.
    1 - Brown cows give chocolate milk.
    2 - Brown cows give white milk.
    3 - White cows do not give white milk.
    4 - White cows give chocolate milk.
    5 - None of these.

16. IF ED IS BUSY, SUE IS TOO, ED IS NOT BUSY.
    1 - Sue is busy.
    2 - Sue is not busy.
    3 - Ed and Sue are friends.
    4 - Ed is busy.
    5 - None of these.

* * * * * * * * * * * * *

PART II: REMEMBER TO ACCEPT THE FIRST SENTENCES AS TRUE. DO NOT PUT ANY MARKS ON YOUR ANSWER SHEET IN THE SPACE PROVIDED FOR QUESTIONS 17-40. BEGIN ANSWERING THE FOLLOWING QUESTIONS ON YOUR ANSWER SHEET IN THE SPACE NUMBERED 41.

41. MY GUN IS RUSTY OR BROKEN.
    1 - If my gun is not rusty, it is broken.
    2 - My gun is rusty.
    3 - My gun is broken.
    4 - I have a rusty broken gun.
    5 - None of these.

42. I HAVE FIVE COINS. EACH OF THEM IS A PENNY.
    1 - I have no pennies.
    2 - I have just one penny.
    3 - One of my coins could be a dime.
    4 - I have five pennies.
    5 - None of these.

43. IF SAILORS SWIM, FISH FRY.
    1 - If sailors do not swim, fish do not fry.
    2 - If fish fry, sailors swim.
    3 - If fish do not fry, sailors do not swim.
    4 - Things that swim are not fish.
    5 - None of these.

44. EVERY TEACHER IS PRETTY.
    1 - Short English teachers are ugly.
    2 - Blond teachers are not pretty.
    3 - My teacher is pretty.
    4 - Arithmetic teachers are not pretty.
    5 - None of these.

45. ENALPS HAVE HORNS AND SLINKS ARE PINK.
    1 - Enalps have pink horns.
    2 - Slinks have pink horns.
    3 - Enalps are pink.
    4 - Slinks are pink.
    5 - None of these.

46. ANY ADDITION PROBLEM IS EASY.
    1 - 2 + 2 is a hard problem.
    2 - 2 + 3 + 4 + 5 is a hard problem.
    3 - 37,981 + 84,277 is an easy problem.
    4 - 3 x 4 is an easy problem.
    5 - None of these.

47. BOB KNOWS THAT 2 + 2 = 4 OR HE CAN SPELL "CAT."
    1 - If Bob does not know that 2 + 2 = 4, then he knows that C A T spells "cat."
    2 - If Bob can't spell "cat," he thinks that 2 + 2 = 22.
    3 - Bob knows that 2 + 2 = 4.
    4 - Bob knows that C A T spells "cat."
    5 - None of these.

48. SOME GIRLS IN THIS ROOM ARE SHORT.
    1 - No girls in this room are tall.
    2 - Some girls in this room are tall.
    3 - Mary is short if she is in this room.
    4 - Ann is in this room, so she is tall.
    5 - None of these.

49. I WILL GO IF YOU STAY.
    1 - If you will not stay, I will not go.
    2 - If I will go, you will stay.
    3 - You will not stay if I will not go.
    4 - I will stay if you go.
    5 - None of these.

50. ALL SAILORS SWIM. ART AND ED ARE SAILORS.
    1 - Ed does not know how to swim.
    2 - Ed can swim better than Art.
    3 - Art cannot swim.
    4 - Art can swim.
    5 - None of these.

51. CURT IS IN THE DIRT AND GUY IS IN THE SKY.
    1 - Curt is in the sky.
    2 - Guy is in the dirt.
    3 - Curt is not in the dirt.
    4 - Guy is in the sky.
    5 - None of these.

52. JIM LOST ALL OF HIS MARBLES.
    1 - Jim now has just one marble.
    2 - Jim now has more than one marble.
    3 - Jim had more than one marble before he lost them.
    4 - Jim now has no marbles.
    5 - None of these.

53. IF ART IS NOT SMART, THEN MARY IS MERRY.
    1 - Mary is smart or Art is merry.
    2 - Art and Mary are both smart.
    3 - Mary is merry and Art is smart.
    4 - Mary is merry or Art is smart.
    5 - None of these.

54. ANN DOES NOT HAVE ANY MONEY.
    1 - Ann has no nickels.
    2 - Ann has at least 1¢.
    3 - Ann has three dimes.
    4 - Ann has two dollar bills.
    5 - None of these.

55. MARY IS NOT SICK IF SHE ATE A PURPLE APPLE.
    1 - If Mary is sick, she did not eat a purple apple.
    2 - If Mary is not sick, she ate a purple apple.
    3 - If Mary did not eat a purple apple, she is sick.
    4 - Mary ate a purple apple.
    5 - None of these.

56. SOME BOYS LIKE GIRLS.
    1 - At least one boy likes girls.
    2 - Jim likes Mary.
    3 - Tony does not like Beth.
    4 - At least one girl likes boys.
    5 - None of these.

57. IT IS MARCH AND SNOW IS FALLING.
    1 - It does not snow in April.
    2 - It is snowing March.
    3 - It is April and snow is falling.
    4 - It is March and snow is not falling.
    5 - None of these.

APPENDIX 3

FINAL TEST INSTRUMENT

DIRECTIONS: THIS IS NOT A TEST OF HOW MUCH YOU KNOW; IT IS A TEST OF HOW YOU THINK. IN EACH TEST ITEM, ONE OR TWO SENTENCES ARE GIVEN IN BOLD PRINT; ACCEPT THE SENTENCES IN BOLD PRINT AS TRUE. THE SENTENCES IN BOLD PRINT ARE FOLLOWED BY FIVE CHOICES WHICH ARE MARKED 1, 2, 3, 4, AND 5. ONE OF THE FIVE IS ALWAYS TRUE BECAUSE OF THE TRUTH OF THE FIRST SENTENCES. ON YOUR ANSWER SHEET, MARK THE NUMBER OF THE CHOICE WHICH YOU ACCEPT AS TRUE. LOOK AT THE FOLLOWING EXAMPLES:

Example 1: I AM A BOY AND I LIKE PEPPER.
1 - I like salt.
2 - I like pepper.
3 - I hate sugar.
4 - I am a girl.
5 - None of these.

The best answer is "2.".

Example 2: IF APPLES ARE PURPLE, THEN PEARS ARE BLUE.
APPLES ARE PURPLE.
1 - Apples are orange.
2 - Apples are white.
3 - Pears are blue.
4 - Pears are purple.
5 - None of these.

Which is the best answer?

Example 3: IF MY PENCIL IS RED, IT HAS AN ERASER.
MY PENCIL HAS AN ERASER.
1 - My pencil is red.
2 - My pencil is not red.
3 - My pencil is yellow.
4 - My pencil is black with pink stripes.
5 - None of these.

What is the best answer?

PART I: DO NOT WRITE ON THIS TEST.

1. I OWN A RED CAR IF I OWN A FORD. I OWN A FORD.
   1 - I do not own a Dodge.
   2 - I own a red car.
   3 - I do not own a Ford.
   4 - I do not own a red car.
   5 - None of these.

2. SUE IS MARRIED IF DALE IS A GIRL. SUE IS MARRIED.
   1 - Dale is a girl.
   2 - Dale is a boy.
   3 - Dale is married.
   4 - Dale is not married.
   5 - None of these.

3. IF THE CAR RUNS, IT HAS GAS. THE CAR DOES NOT RUN.
   1 - The car is red.
   2 - The car does not have gas.
   3 - Nobody started the car.
   4 - The car has a flat tire.
   5 - None of these.

4. IF JIM IS A BOY, THEN JIM IS GOOD LOOKING. JIM IS NOT GOOD LOOKING.
   1 - Jim is not a girl.
   2 - Jim is not a boy.
   3 - Jim is not a woman.
   4 - Jim is not a man.
   5 - None of these.

5. THE CAR IS BLUE IF THE TRUCK IS GREEN. THE CAR IS NOT BLUE.
   1 - The car is blue.
   2 - The car is green.
   3 - The truck is green.
   4 - The truck is not green.
   5 - None of these.

6. IF THE LIGHT IS GREEN, I'LL CROSS THE STREET. I'LL CROSS THE STREET.
   1 - The light is green.
   2 - The light is yellow.
   3 - The light is red.
   4 - The light is burned out.
   5 - None of these.

7. IF IRON IS STRONG, IT IS GRAY. IRON IS NOT STRONG.
   1 - Iron is strong.
   2 - Iron is gray.
   3 - Iron is not gray.
   4 - Iron is soft.
   5 - None of these.

8. IF JOE IS HANDSOME, THEN I AM A MONKEY'S UNCLE. JOE IS HANDSOME.
   1 - I am handsome.
   2 - Joe is a monkey's uncle.
   3 - I am not a monkey's uncle.
   4 - I am a monkey's uncle.
   5 - None of these.

9. IF JANE IS PRETTY, MEN WILL LOOK AT HER. MEN LOOK AT JANE.
   1 - Jane is ugly.
   2 - Jane is pretty.
   3 - Jane is smart.
   4 - Jane has on a mini-skirt.
   5 - None of these.

10. IF THE WIND BLOWS IN MAY, IT WILL BE FOGGY IN APRIL. THE WIND BLOWS IN MAY.
   1 - It will not be foggy in April.
   2 - It will not be foggy in May.
   3 - It will be foggy in April.
   4 - It will be foggy in May.
   5 - None of these.

11. IF MR. JONES IS OLD, HE HAS GRAY HAIR. MR. JONES DOES NOT HAVE GRAY HAIR.
   1 - Mr. Jones is not old.
   2 - Mr. Jones is bald.
   3 - Mr. Jones has brown hair.
   4 - Mr. Jones is old.
   5 - None of these.

12. IF IT RAINS, I'LL BE THERE. IT DOES NOT RAIN.
   1 - I won't be there.
   2 - I'll get wet.
   3 - I'll be there.
   4 - I need an umbrella.
   5 - None of these.

13. IF I DRIVE A CAR, I NEED A DRIVER'S LICENSE. I DON'T NEED A DRIVER'S LICENSE.
   1 - I drive a truck.
   2 - I drive a car.
   3 - I need a driver's license.
   4 - I don't drive a car.
   5 - None of these.

14. IF JIM READS BOOKS, HE WILL BE SMART. JIM READS BOOKS.
   1 - Jim will be smart.
   2 - Smart people read books.
   3 - Jim will not be smart.
   4 - Jim will not read books.
   5 - None of these.

15. IF BROWN COWS GIVE CHOCOLATE MILK, THEN WHITE COWS GIVE WHITE MILK. WHITE COWS GIVE WHITE MILK.
    1 - Brown cows give chocolate milk.
    2 - White cows do not give white milk.
    3 - Brown cows give white milk.
    4 - White cows give chocolate milk.
    5 - None of these.

16. IF ED IS BUSY, SUE IS TOO. ED IS NOT BUSY.
    1 - Sue is busy.
    2 - Sue is not busy.
    3 - Ed and Sue are friends.
    4 - Ed is busy.
    5 - None of these.

PART II: REMEMBER TO ACCEPT THE SENTENCES IN BOLD PRINT AS TRUE. DO NOT PUT ANY MARKS ON YOUR ANSWER SHEET IN THE SPACES FOR ITEMS 17-40. BEGIN ANSWERING THE FOLLOWING QUESTIONS ON YOUR ANSWER SHEET IN THE SPACE NUMBERED 41.

41. MY GUN IS RUSTY OR BROKEN.
    1 - If my gun is not rusty, it is broken.
    2 - My gun is rusty.
    3 - My gun is broken.
    4 - I have a rusty broken gun.
    5 - None of these.

42. I HAVE FIVE COINS. EACH OF THEM IS A PENNY.
    1 - One of my coins could be a dime.
    2 - I have no pennies.
    3 - I have just one penny.
    4 - I have five pennies.
    5 - None of these.

43. IF SAILORS SWIM, FISH FRY.
    1 - If sailors do not swim, fish do not fry.
    2 - If fish fry, sailors swim.
    3 - If fish do not fry, sailors do not swim.
    4 - Things that swim are not fish.
    5 - None of these.

44. EVERY TEACHER IS PRETTY.
    1 - Blond teachers are not pretty.
    2 - Short English teachers are ugly.
    3 - My teacher is pretty.
    4 - Arithmetic teachers are not pretty.
    5 - None of these.

45. ENALPS HAVE HORNS AND SLINKS ARE PINK.
    1 - Slinks have pink horns.
    2 - Enalps have pink horns.
    3 - Enalps are pink.
    4 - Slinks are pink.
    5 - None of these.

46. ANY ADDITION PROBLEM IS EASY.
    1 - 2 + 2 is a hard problem.
    2 - 2 + 3 + 4 + 5 is a hard problem.
    3 - 37,981 + 84,277 is an easy problem.
    4 - 3 x 4 is an easy problem.
    5 - None of these.

47. BOB KNOWS THAT 2 + 2 = 4 OR HE CAN SPELL "CAT."
    1 - If Bob does not know that 2 + 2 = 4, then he knows that C A T spells "cat."
    2 - If Bob can't spell "cat," he thinks that 2 + 2 = 22.
    3 - Bob knows that 2 + 2 = 4.
    4 - Bob knows that C A T spells "cat."
    5 - None of these.

48. SOME GIRLS IN THIS ROOM ARE SHORT.
    1 - No girls in this room are tall.
    2 - Some girls in this room are tall.
    3 - Mary is short if she is in this room.
    4 - Ann is in this room, so she is tall.
    5 - None of these.

49. IF MEN WEAR BELTS, THEY FIGHT FIRES.
    1 - If men do not wear belts, they do not fight fires.
    2 - Men do not fight fires if they wear belts.
    3 - Men wear belts if they do not fight fires.
    4 - If men fight fires, they wear belts.
    5 - None of these.

50. ALL SAILORS SWIM. ART AND ED ARE SAILORS.
    1 - Ed does not know how to swim.
    2 - Ed can swim better than Art.
    3 - Art cannot swim.
    4 - Art can swim.
    5 - None of these.

51. CURT IS IN THE DIRT AND GUY IS IN THE SKY.
    1 - Curt is in the sky.
    2 - Guy is in the dirt.
    3 - Curt is not in the dirt.
    4 - Guy is in the sky.
    5 - None of these.

52. JIM LOST ALL OF HIS MARBLES.
    1 - Jim had more than one marble before he lost them.
    2 - Jim now has just one marble.
    3 - Jim now has more than one marble.
    4 - Jim now has no marbles.
    5 - None of these.

53. IF ART IS NOT SMART, THEN MARY IS MERRY.
    1 - Mary is smart or Art is merry.
    2 - Art and Mary are both smart.
    3 - Mary is merry and Art is smart.
    4 - Mary is merry or Art is smart.
    5 - None of these.

54. ANN DOES NOT HAVE ANY MONEY.
    1 - Ann has no nickels.
    2 - Ann has three dimes.
    3 - Ann has at least 1¢.
    4 - Ann has two dollar bills.
    5 - None of these.

55. MARY IS NOT SICK IF SHE ATE A PURPLE APPLE.
    1 - If Mary is sick, she did not eat a purple apple.
    2 - If Mary is not sick, she ate a purple apple.
    3 - If Mary did not eat a purple apple, she is sick.
    4 - Mary ate a purple apple.
    5 - None of these.

56. SOME BOYS LIKE GIRLS.
    1 - At least one boy likes girls.
    2 - Jim likes Mary.
    3 - Tony does not like Beth.
    4 - At least one girl likes boys.
    5 - None of these.

57. IT IS MARCH AND SNOW IS FALLING.
    1 - It does not snow in April.
    2 - It is snowing in March.
    3 - It is April and snow is falling.
    4 - It is March and snow is not falling.
    5 - None of these.

APPENDIX 4

COMMUNICATION WITH ORANGE COUNTY SCHOOL ADMINISTRATION

Box 12963 University Station
Gainesville, Florida 32601
December 10, 1968

Mr. William Hadlock
Orange County Public School
Administration Building
North Tampa Avenue
Orlando, Florida

Dear Mr. Hadlock,

The ability of students to think well is very likely a prime objective of most education. In studying any discipline of knowledge, we are primarily concerned with the type of thinking done in the discipline. In mathematics, for example, logical thinking is a prime concern.

A starting point for studying the logical thinking done in mathematics can be to find out when students understand some types of logic used in mathematics. I am presently working on a doctoral dissertation at the University of Florida to determine when students naturally develop the abilities to understand selected types of logic. Jean Piaget, Shirley Hill (Stanford University, 1961), and Robert Ennis (Cornell University, 1961) have done similar work with logic.

My study is a survey. In order to find students' abilities, I have constructed a test of the selected concepts of logic which I have chosen and hope to administer it myself to approximately 1100 fourth through twelfth grade students some time in February. The test will take no more than fifty minutes.

My doctoral committee, chaired by Dr. Dwain E. Small, has suggested that the Orange County School System would offer the best cross section of student population and also has an administration which is interested in the upgrading of education. Hence, I am requesting permission to use the schools of Orange County for my sample.

I want to test fourth through twelfth grade students (a total of 39 classes). The test should be administered to a cross section of the students of the school system; students of high, medium, and low socioeconomic status need to be included in each grade level tested. I also need permission to look at school records to get I.Q. test scores, scores on tests of level of achievement of reading, sex (when necessary), and parent's occupation. I would like to begin testing on February 3, 1969, or as soon afterward as possible.

Will you use the enclosed stamped, self-addressed envelope to indicate whether or not I may use the Orange County School System for my sample? It will be possible for me to come to Orlando to discuss this matter with the appropriate school official whom you wish to designate. Dr. Small has indicated to me that he would be willing to correspond with you on this matter if you feel this is necessary.

Sincerely yours,

Robert S. Matulis

APPENDIX 5

NAMES OF SCHOOLS USED IN THIS STUDY

The following Orange County, Florida, schools were used in this study:

Edgewater High School

Evans High School

Fern Creek Elementary School

Grand Avenue Elementary School

Jones Junior and Senior High School

Lake Silver Elementary School

Maitland Junior High School

Memorial Junior High School

All of the schools are in Orlando except Maitland Junior High School, which is in Maitland, a suburb of Orlando.

APPENDIX 6

ORAL DIRECTIONS GIVEN IN ADMINISTERING THE TEST

ORAL TEST DIRECTIONS

The test which you are about to take is part of a survey. It will find out when people have the greatest development in their ability to think. Over 1000 fourth through twelfth graders in Orlando will be given the same test. Scores will be studied to find out at which age people improve the most in their ability to think.

Questions are hard. No twelfth-grade students got all questions right so far. Don't give up. Do what you can. Some items are easier than others. If you have trouble with an item, try the next one. There is no penalty for guessing, so make a good guess if you are not sure of the answer to a question.

The test is made up of situations which are not necessarily real. Do not answer any questions on the basis of what you know. In each test item, one or two sentences will be given; they will be printed in bold print. You must accept the sentences in bold print as true; pretend they are true; make believe they are true. These sentences will be followed by five choices. One of the five will be true; it will be the correct answer. The correct answer to each question, in other words, is the answer which is reasonable when we accept the first sentences in bold print.

(Each student was now given one test, one answer sheet, one 5" x 8" card, and one number two pencil.)

Does everybody have a test, an answer sheet, a 5" x 8" card, and a pencil? Do not write on the test. Put only your answers on the answer sheet. Now take the 5" x 8" card. Put your name on it in the top blank.

(Pause)

Find the number on your answer sheet. It is printed in black. Put this number in the second blank on your 5" x 8" card.

(Pause)

Now read the directions to the test as I read them aloud. DIRECTIONS: This is not a test of how much you know; it is a test of how you think. In each test item, one or two sentences are given in bold print which you are to accept as true.

Pretend they are true. These sentences in bold print are followed by five choices which are marked 1, 2, 3, 4, and 5. One of the choices is true always because of the truth of the first sentences in bold print. On your answer sheet, mark the letter of the choice which you accept as true. Look at the following examples now and mark your answers in the spaces on the 5" x 8" card. Completely fill in the space between the correct pair of dotted lines. Make a dark line between them by going back and forth between them with your pencil. Don't make any marks outside the dotted lines.

Example 1: I AM A BOY AND I LIKE PEPPER.
1 - I like salt.
2 - I like pepper.
3 - I hate sugar.
4 - I am a girl.
5 - None of these.

To answer this question, make believe it is true that "I am a boy and I like pepper." Accept it even if you are a girl. If you accept it, you know that the fourth choice is not correct since I am not a girl. Since there is no way to tell from the first sentence if I like salt or hate sugar, the first and third choices are not right. Choice 2 is correct since we assumed that I like pepper in the first sentence. Put a line between the two dotted lines under the "2" in the space for your answer to Example 1 on the 5" x 8" card.

(A cardboard-and-acetate demonstration card which had been correctly marked, was now shown.)

Example 2: IF APPLES ARE PURPLE, THEN PEARS ARE BLUE. APPLES ARE PURPLE.
1 - Apples are orange.
2 - Apples are white.
3 - Pears are blue.
4 - Pears are purple.
5 - None of these.

Make believe it is true that "If apples are purple, then pears are blue." Then anytime we know that apples are purple, we know that pears are blue. We know that apples are purple by the second sentence in bold print, which we are to accept as true. Therefore, we know that pears are blue. The correct answer is "3." All other answers are not correct since, when we pretend that the first sentences are true, it cannot be shown that any of the other choices are true. Put your answer in the right place on the 5" x 8" card.

Let's look at

Example 3: IF MY PENCIL IS RED, IT HAS AN ERASER. MY PENCIL HAS AN ERASER.
1 - My pencil is red.
2 - My pencil is not red.
3 - My pencil is yellow.
4 - My pencil is black with pink stripes.
5 - None of these.

Make believe it is true that "If my pencil is red, it has an eraser." This means that when my pencil is red, I can say it has an eraser. My red pencils have erasers. This does not say anything about my yellow pencils; maybe they have erasers, too. I don't know. Maybe they do; maybe they don't. If a pencil has an eraser, I have no way of knowing what color it is. Red tells me that my pencil has an eraser, but eraser does not tell me that my pencil is red, is not red, or is any color. Therefore the correct answer is "5." Mark it on your 5" x 8" card.

Remember that the correct answer is the one which makes sense when you pretend that the first sentences in bold print are true. The first sentences are the only things to consider in answering questions.

Notice that your answers go across the page on your answer sheet. Question 2 is beside, not below question 1. Answer the first sixteen questions in the spaces on the answer sheet. Do not make any marks in the spaces for answers 17-40. Then answer questions 41-57 in the places on your answer sheet which are labelled 41-57.

Papers will be collected when everyone is done; there will be no time limit on the test. If you finish early, check your answers and wait quietly for everyone else to finish.

Good luck!

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BIOGRAPHICAL SKETCH

Robert Stanley Matulis was born on November 18, 1941, in Milwaukee, Wisconsin, where he lived through 1963. He received his Bachelor of Science Degree in mathematics at Marquette University in June, 1963. He participated in a National Science Foundation Summer Institute in General Science at Eastern Michigan University in 1964, and attended summer sessions at Marquette University in 1965 and 1966. After participating in a National Science Foundation Academic Year Institute in Earth Science, he received his Master of Education Degree in mathematics education at the University of Florida in June, 1967.

Mr. Matulis taught ninth-grade mathematics and tenth-grade general science in Kenosha, Wisconsin, from 1963 to 1966. While pursuing his doctoral program at the University of Florida, he was employed by the University in teaching off-campus mathematics courses to junior and senior high school teachers of mathematics in Jacksonville and Orlando, Florida.

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This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Education and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Education.

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